CS11313 - **Spring** 2022

Design & Analysis of Algorithms

Heapsort

Ibrahim Albluwi

Selection Sort?

```
SELECTION-SORT(array)

for i=n-1 ⟶ 1:
    max = FIND-MAX(array, i, 0)
    swap(array[i], array[max])
```

scan this portion of the array *linearly*.

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prepare(array)

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rearrange the elements so that finding the max is easy!

find the maximum element quickly!

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Roadmap.

1. Review Max-Priority Queues and Heaps.
2. Learn about Heapsort.

Max-Priority Queue (Abstract Data Type)

Abstract Data Type (ADT): A specification of the possible operations on a set of values (independent of the implementation).

Examples.

ADT	Goal	operations
Stack	Remove the item most-recently added	PUSH, POP
Queue	Remove the item least-recently added	ENQUEUE, DEQUEUE

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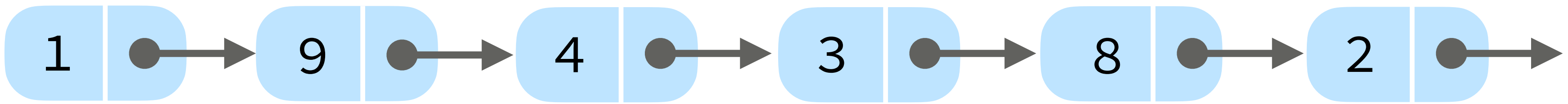
Max-PQ	**Remove the largest item**	INSERT, MAX, DEL-MAX	?

Max-Priority Queue (Abstract Data Type)

Unordered List:



Max-Priority Queue (Abstract Data Type)

Unordered List:

- `insert`: $\Theta(1)$ (insert to the end of the list; order does not matter)
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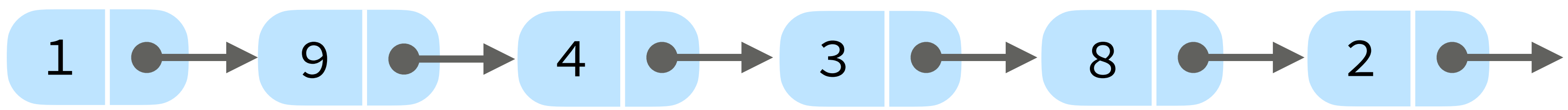

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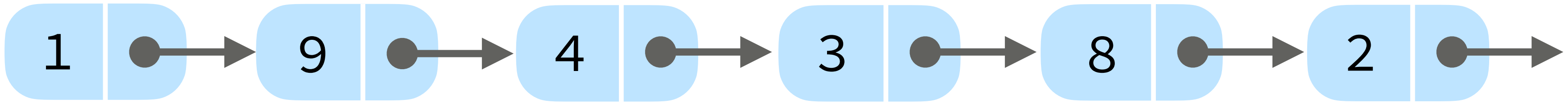

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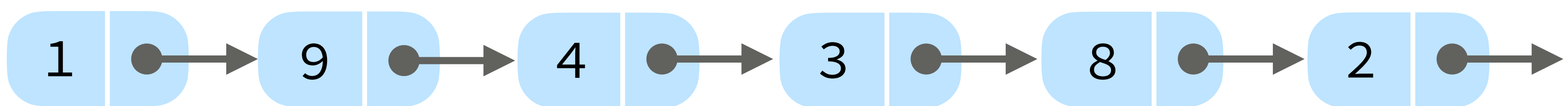

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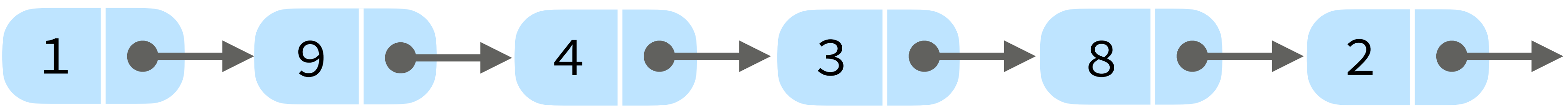

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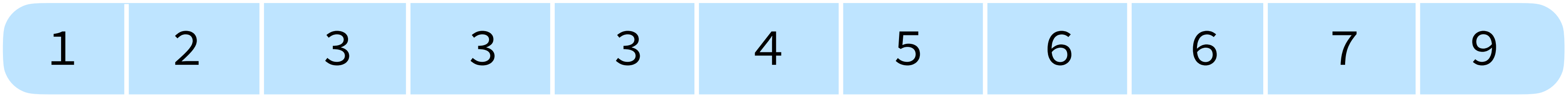

Binary Heap:

- `insert`: $O(\log n)$ (how?)
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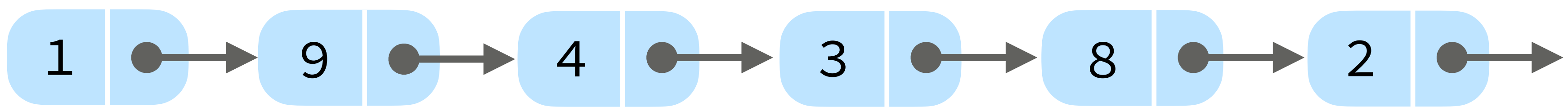

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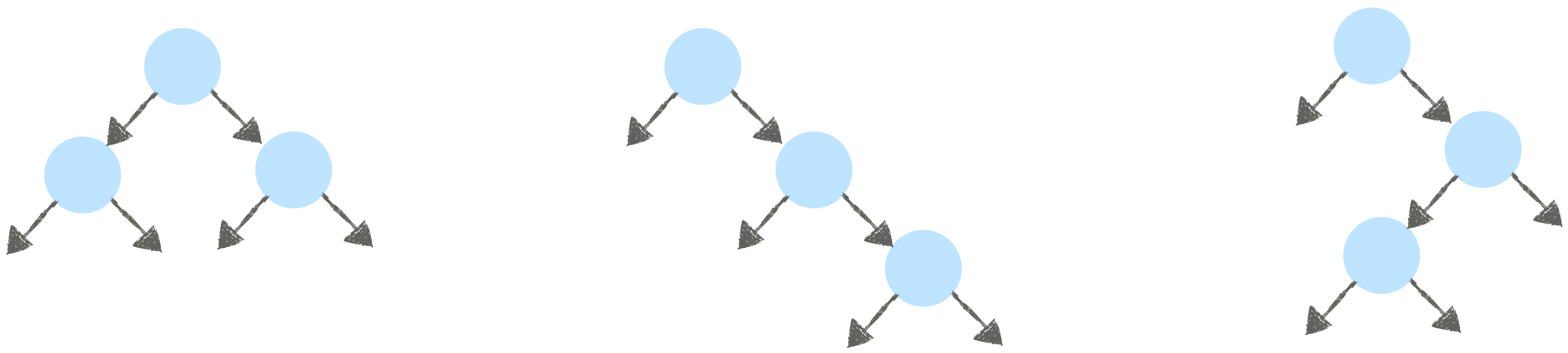
Binary Trees

Binary Tree: Empty or a node with links to left and right binary trees.



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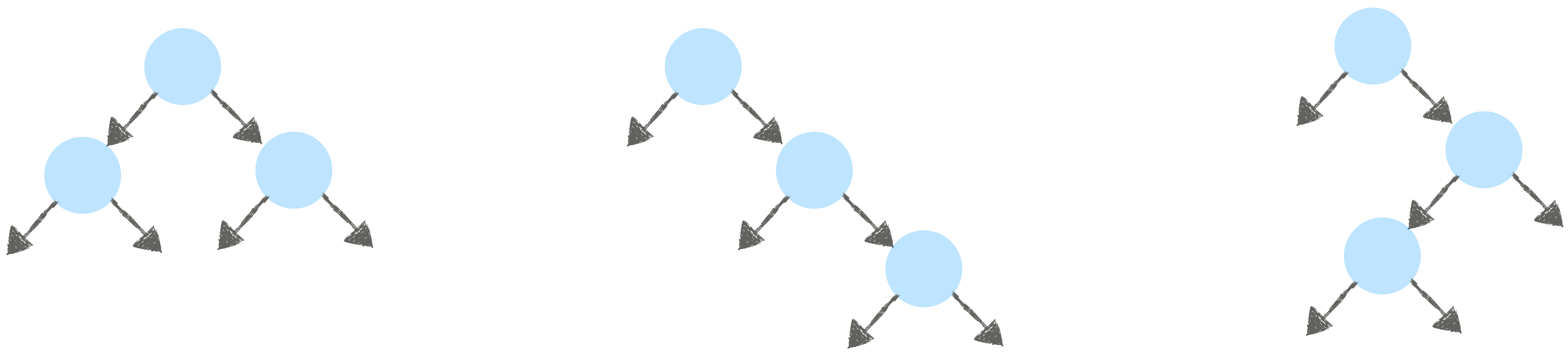
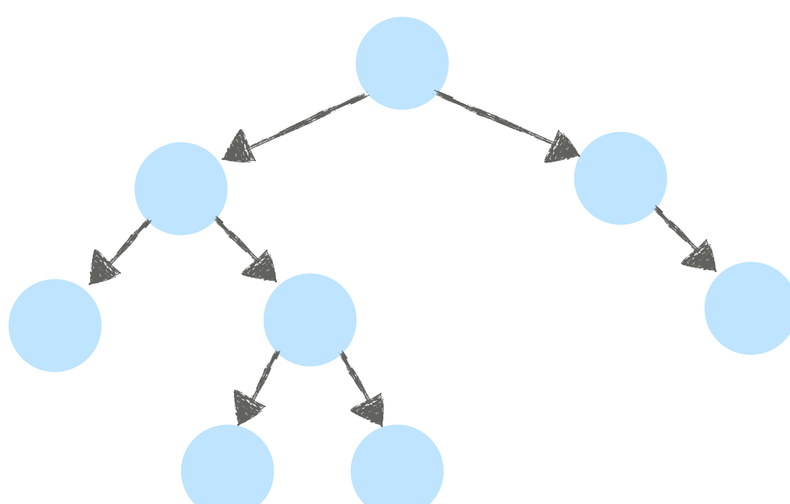
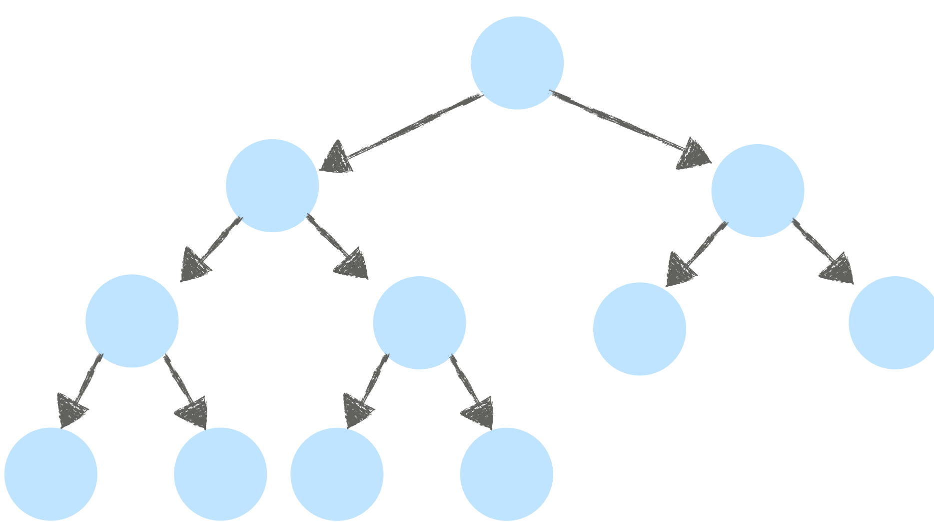


Complete Binary Tree:

- All levels are full (except possibly the last level).
- Last level is filled left-to-right.



not complete



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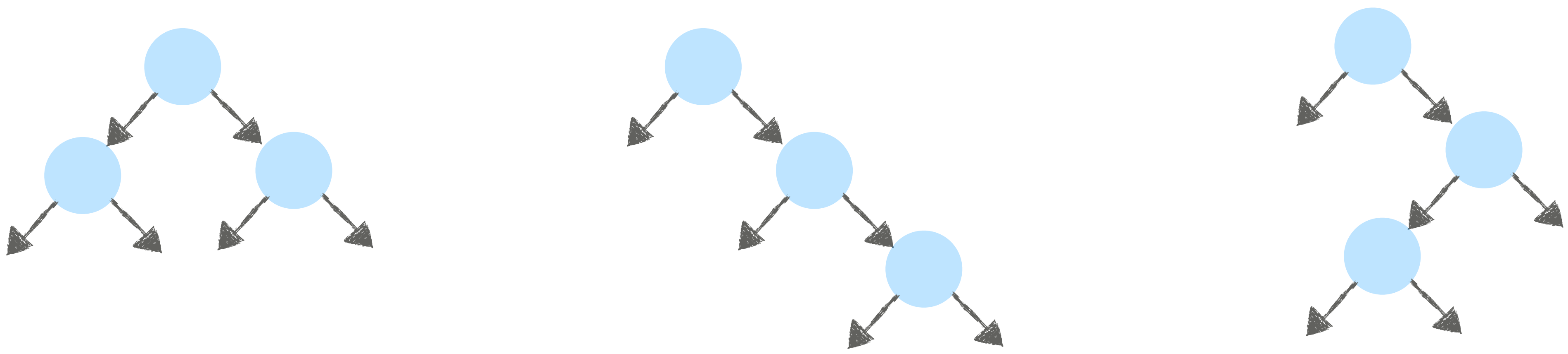

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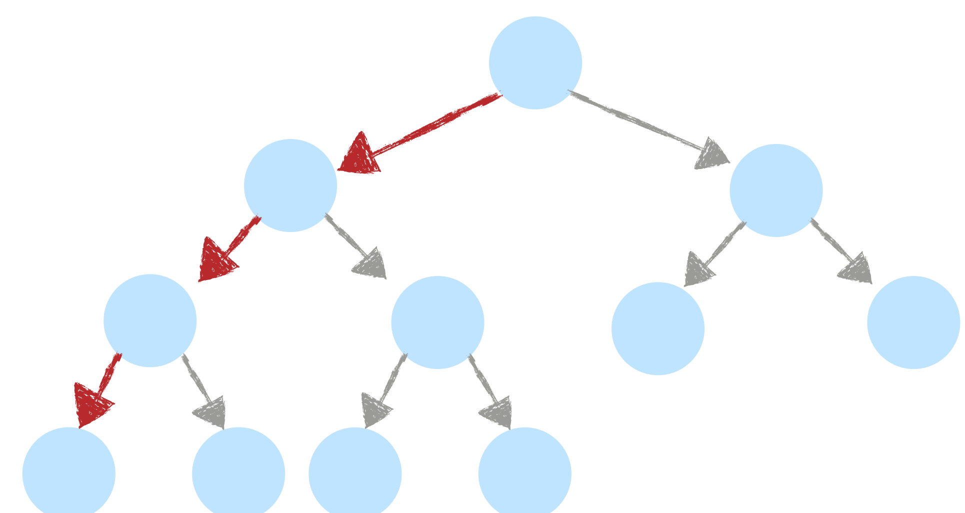

$$h = \lfloor \log_2 11 \rfloor = \lfloor 3.459 \rfloor = 3$$

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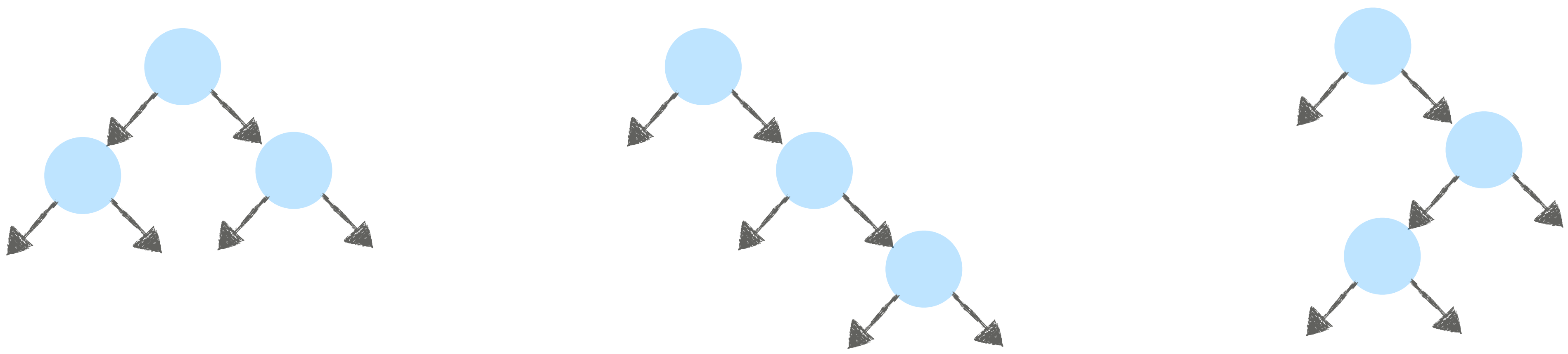

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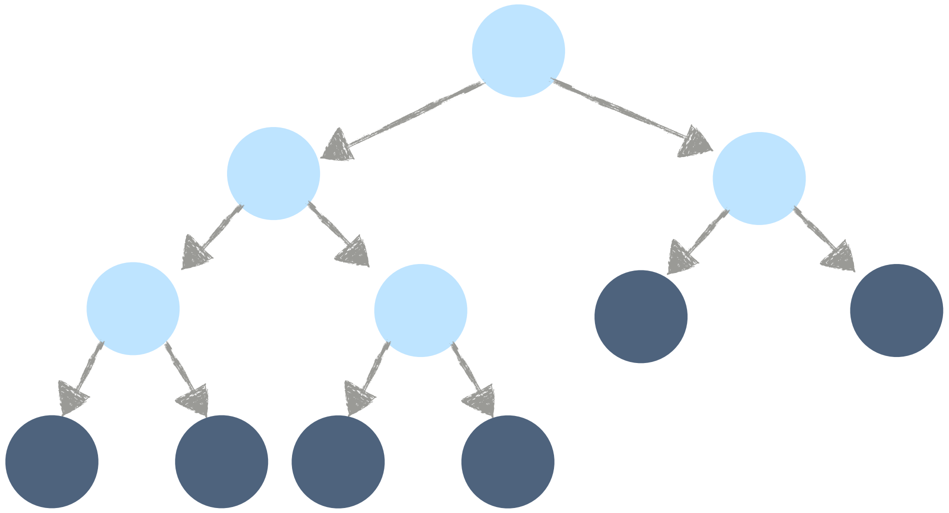

$$\left\lfloor \frac{11+1}{2} \right\rfloor = 6$$

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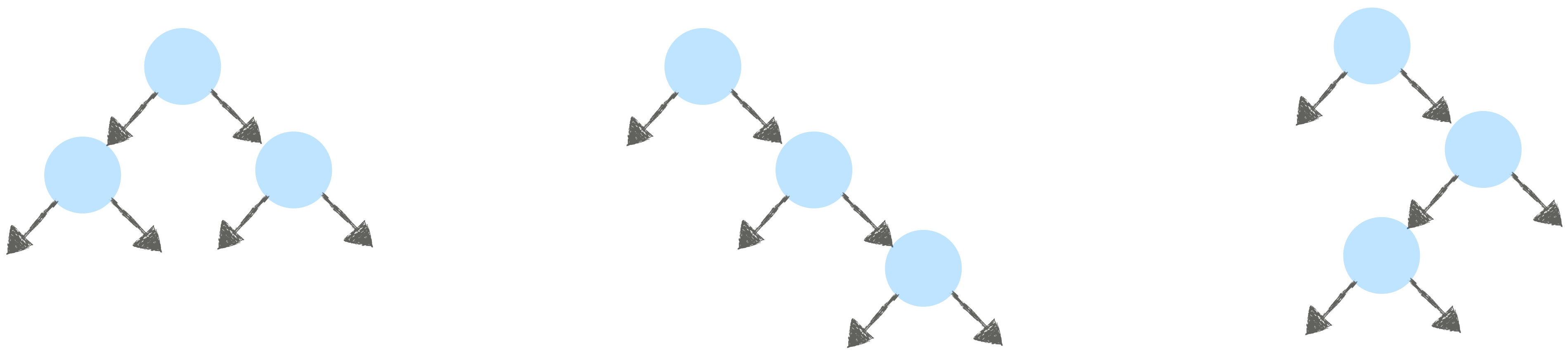

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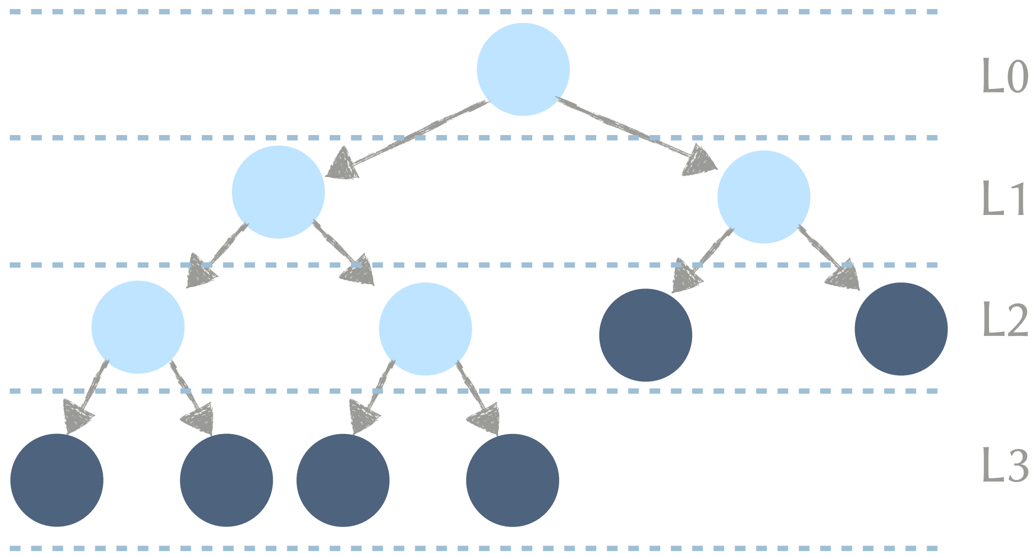

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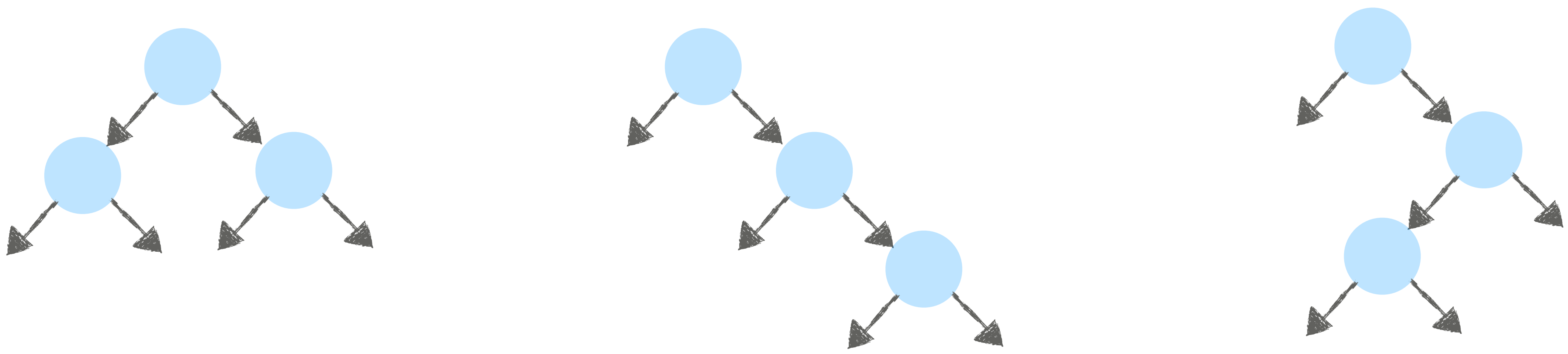

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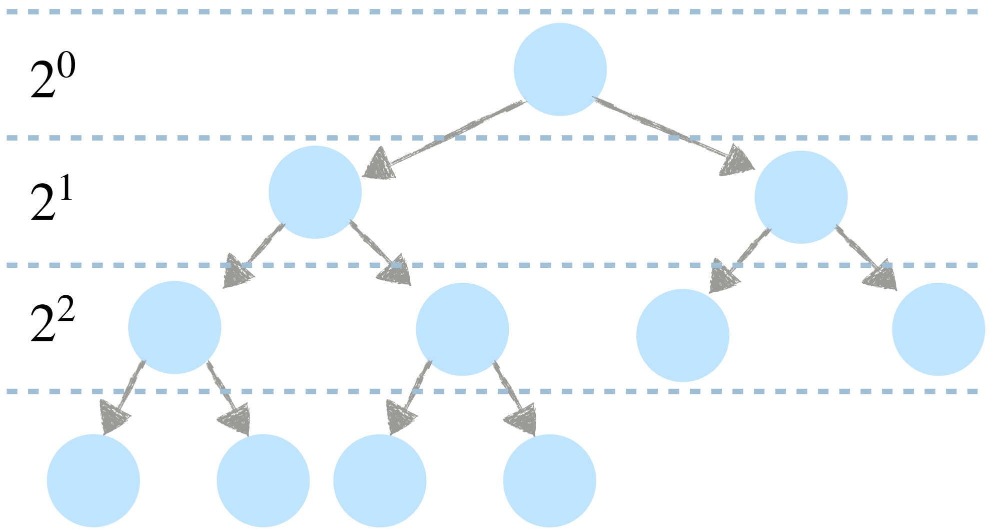
- Number of nodes at *internal* level $i = 2^i$



Binary Heaps (Tree Representation)

Binary Heap: (max-ordered)

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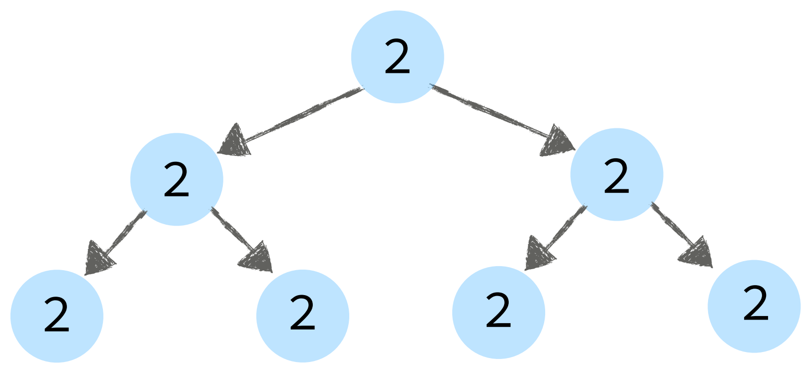

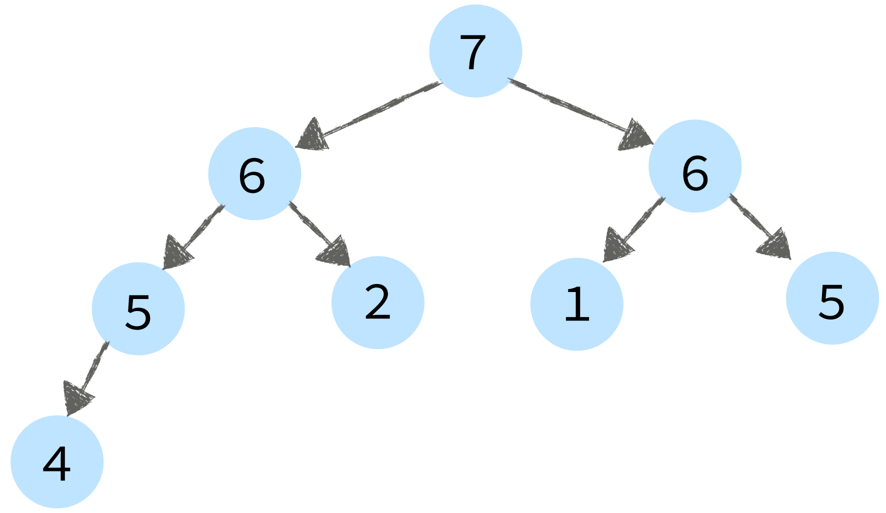

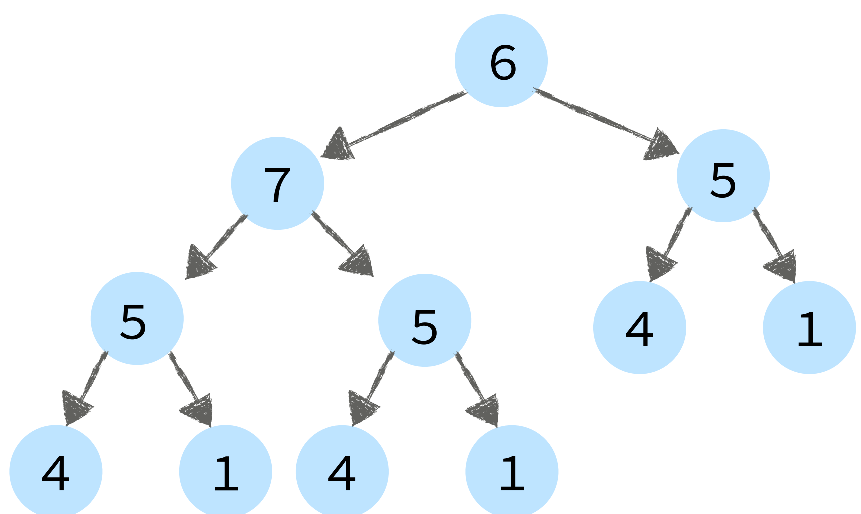

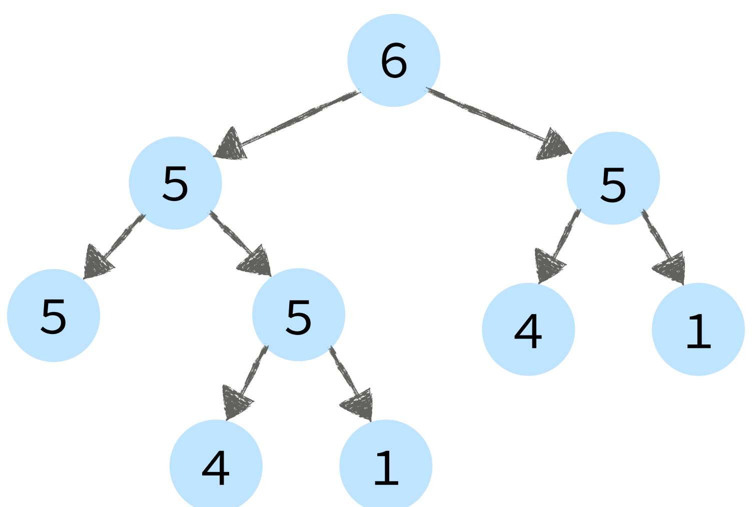

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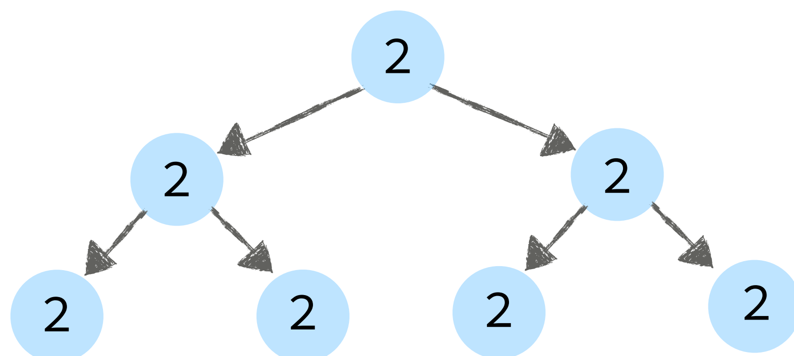

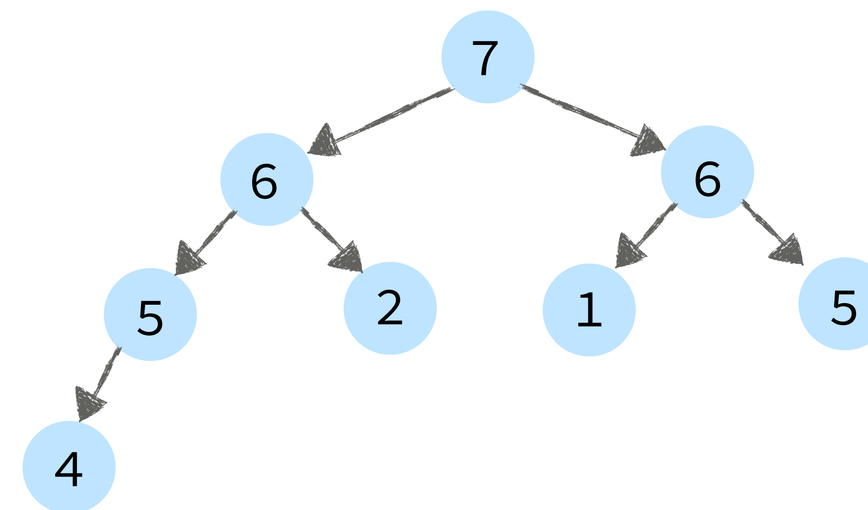

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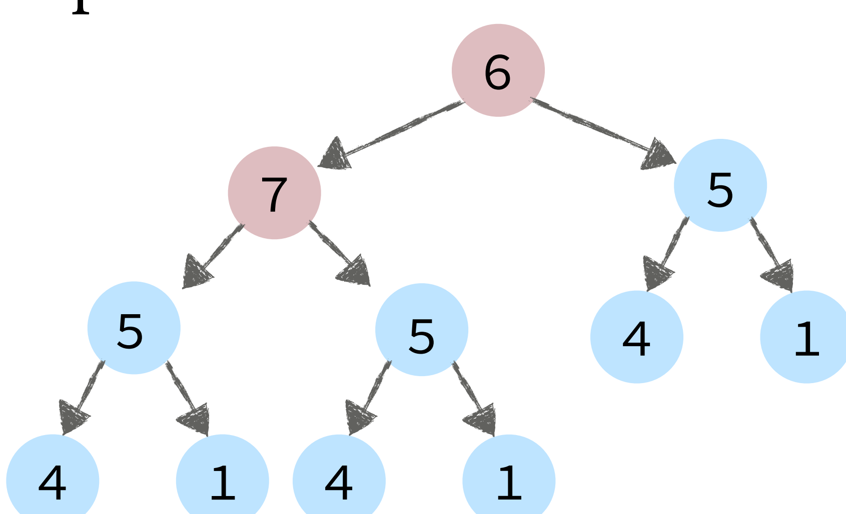
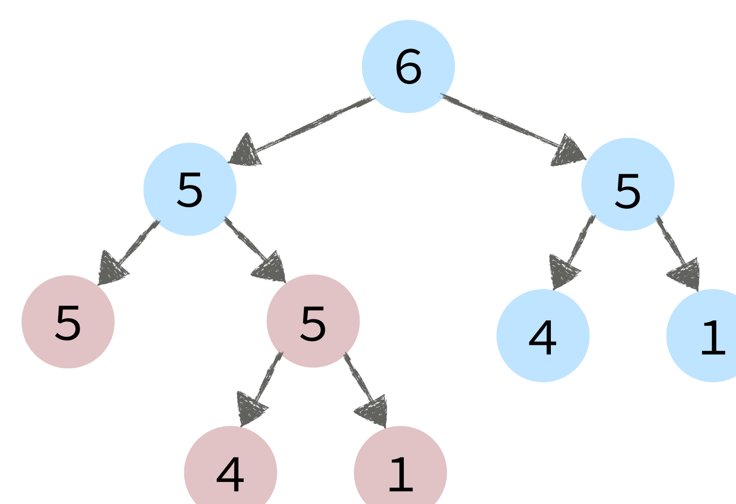


order property violated



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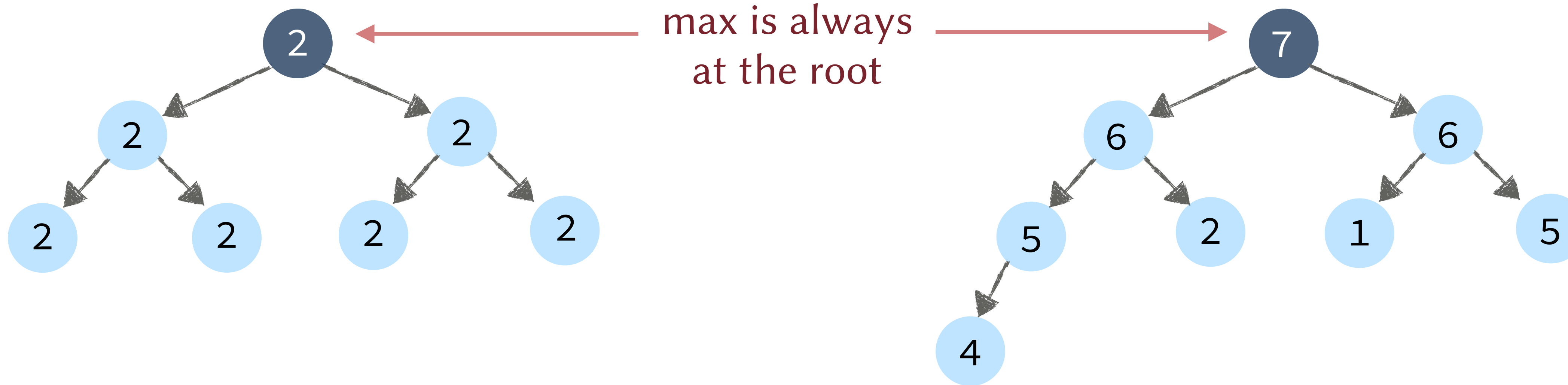

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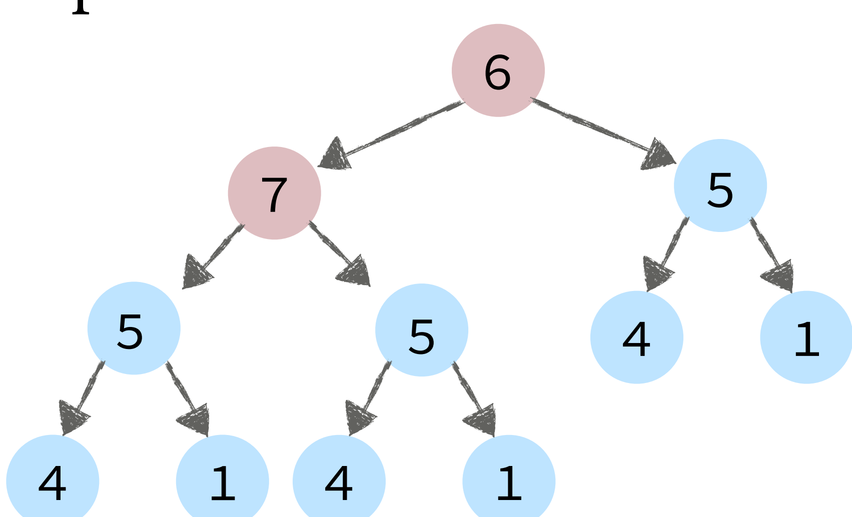

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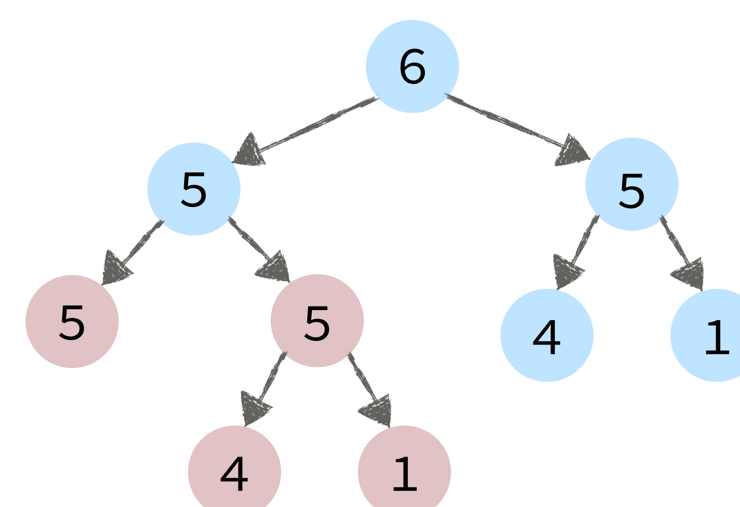

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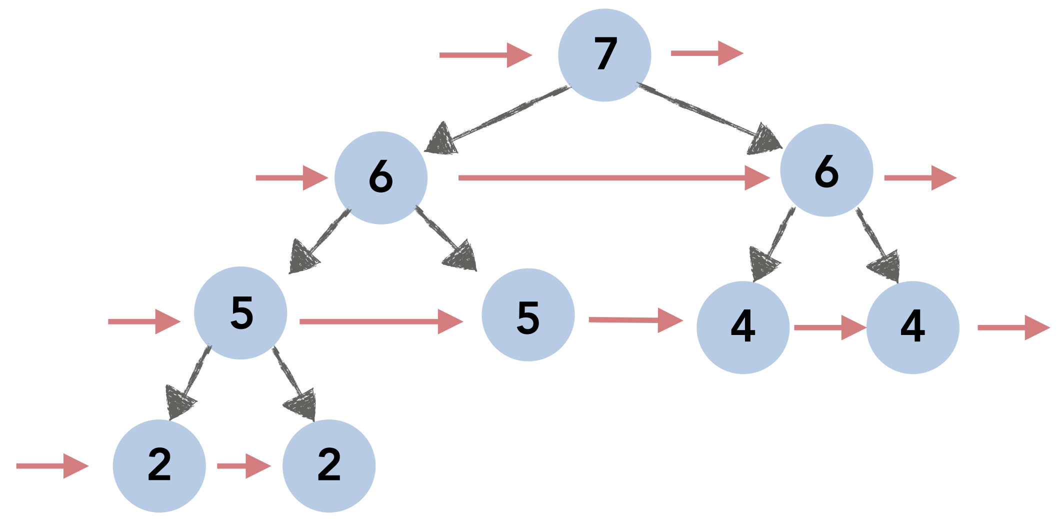
0	1	2	3	4	5	6	7	8
7	6	6	5	5	4	4	2	2

array has the tree
nodes in level-order



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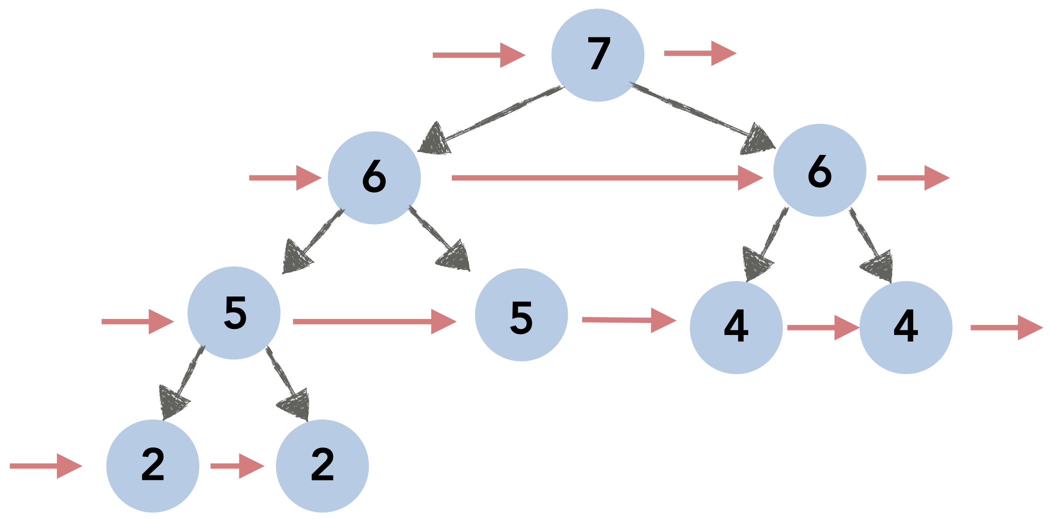

Three simple functions.

LEFT(i)

```
return 2*i + 1
```

RIGHT(i)

```
return 2*i + 2
```

PARENT(i)

```
return (i-1)/2
```

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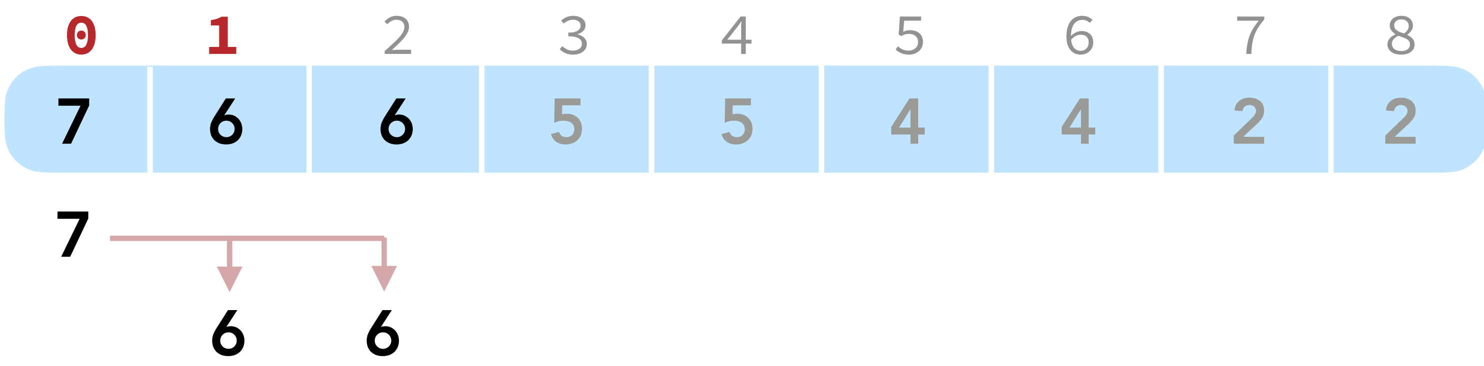

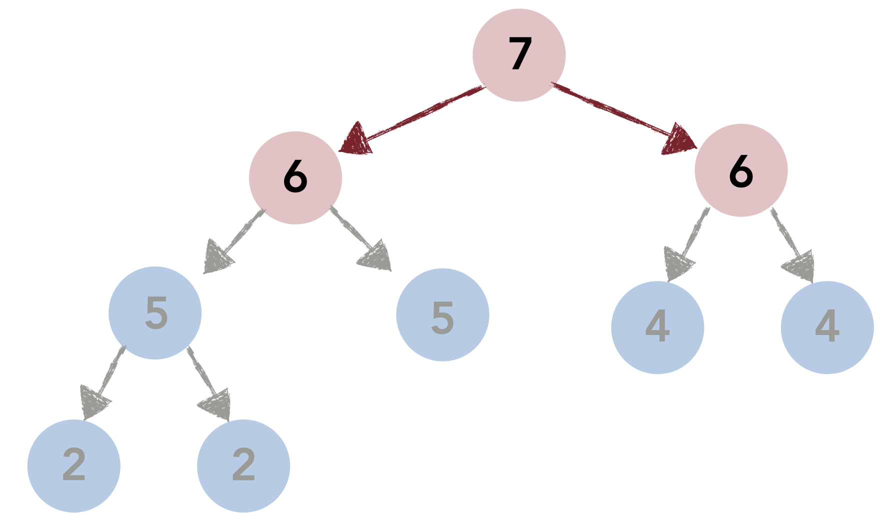

Three simple functions.

LEFT(i)

```
return 2*i + 1
```

left child is at index
2*0 + 1 = 1

RIGHT(i)

```
return 2*i + 2
```

Right child is at index
2*0 + 2 = 2

PARENT(i)

```
return (i-1)/2
```

Parent of the node at 0
is negative (no parent)

Binary Heaps (Array Representation)

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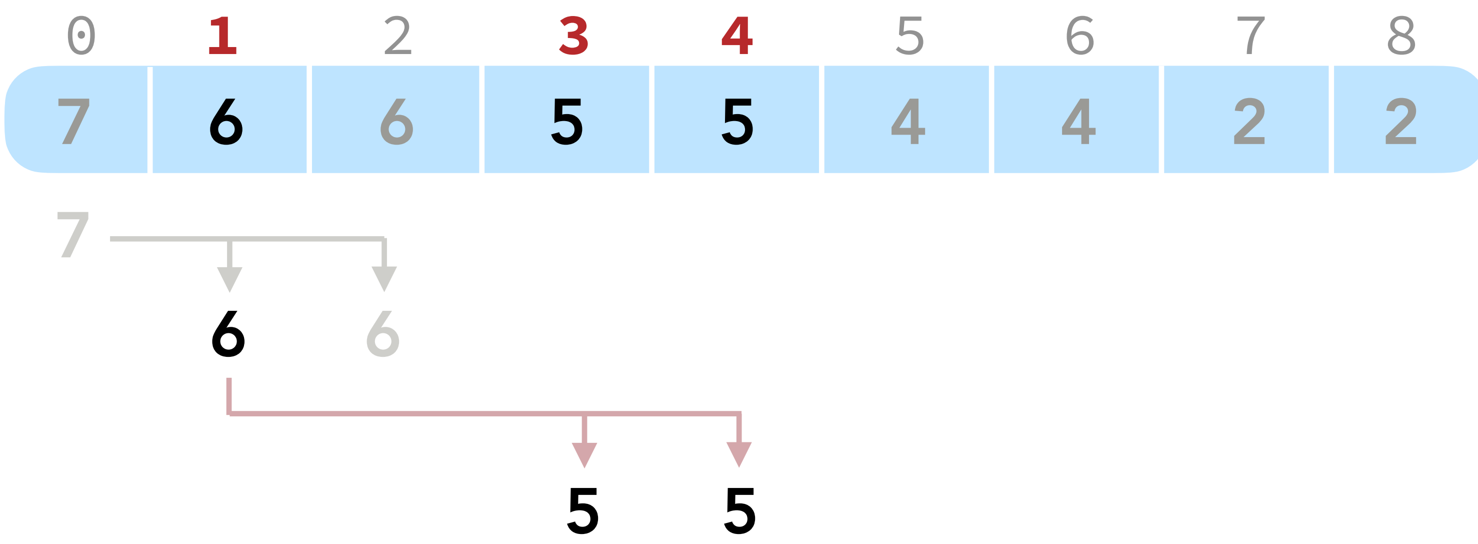

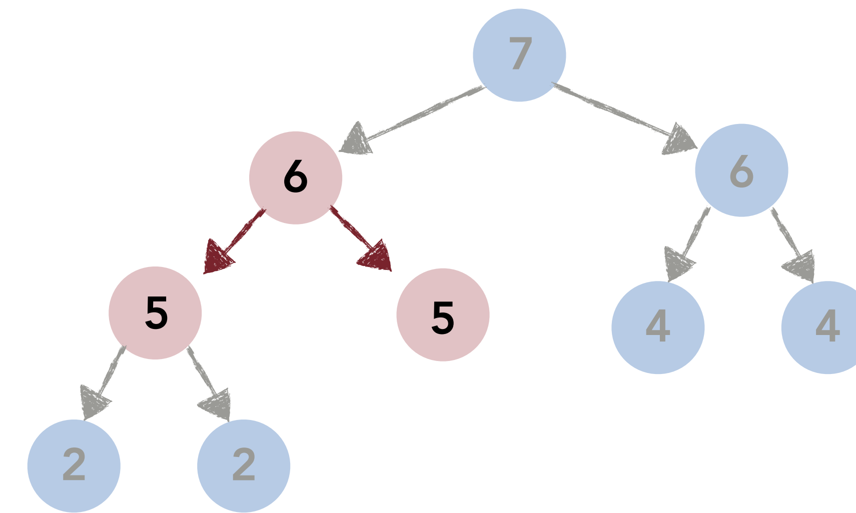

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```
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left child is at index
2*1 + 1 = 3

RIGHT(i)

```
return 2*i + 2
```

Right child is at index
2*1 + 2 = 4

PARENT(i)

```
return (i-1)/2
```

Parent is at index
(1-1)/2 = 0

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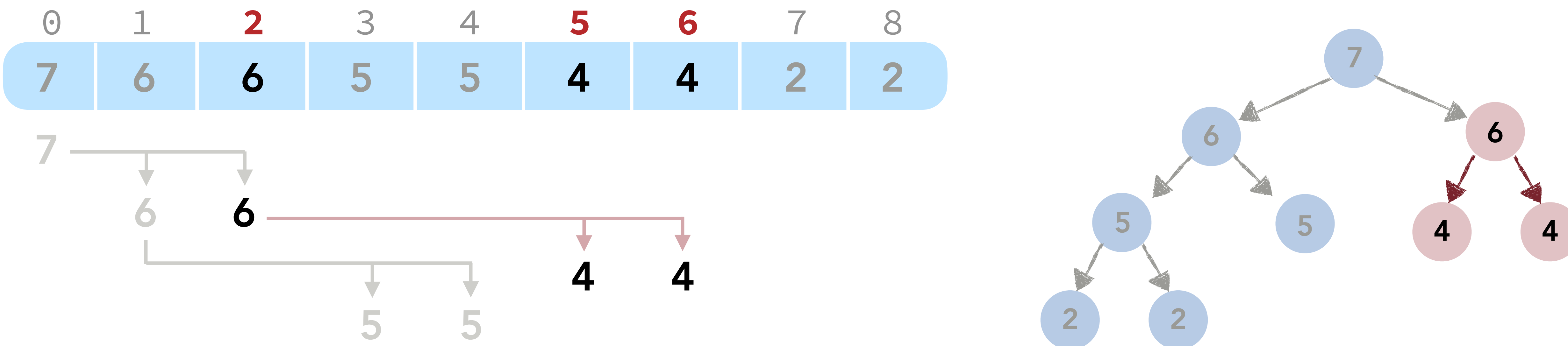

Three simple functions.

LEFT(i)

```
return 2*i + 1
```

left child is at index
2*2 + 1 = 5

RIGHT(i)

```
return 2*i + 2
```

Right child is at index
2*2 + 2 = 6

PARENT(i)

```
return (i-1)/2
```

Parent is at index
(2-1)/2 = 0

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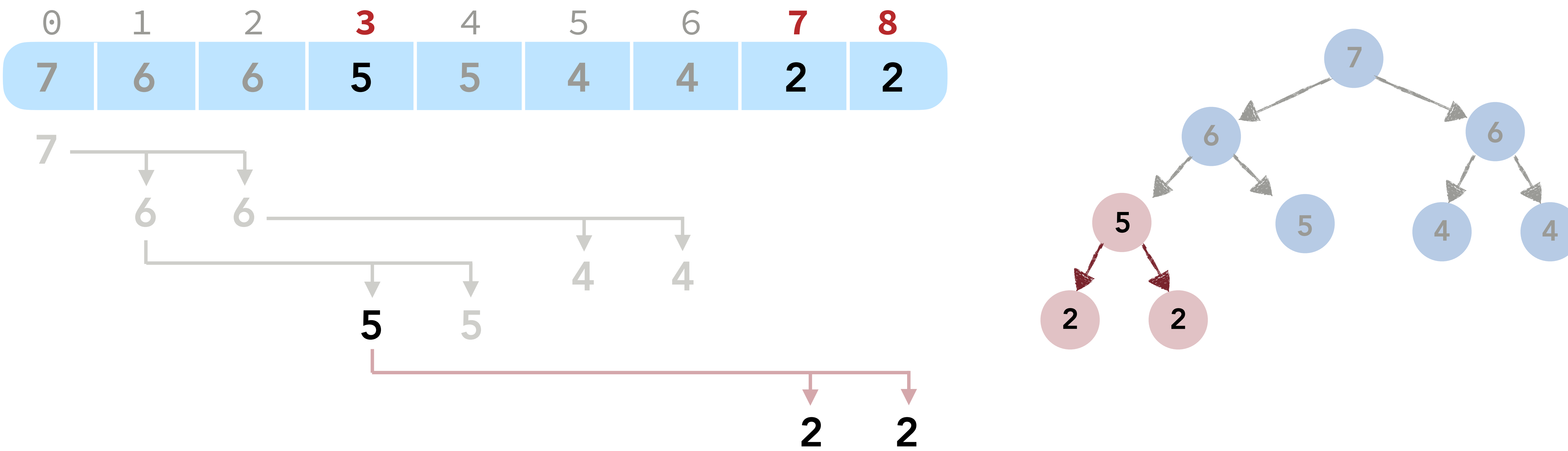

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return 2*i + 1
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left child is at index
2*3 + 1 = 7

RIGHT(i)

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return 2*i + 2
```

Right child is at index
2*3 + 2 = 8

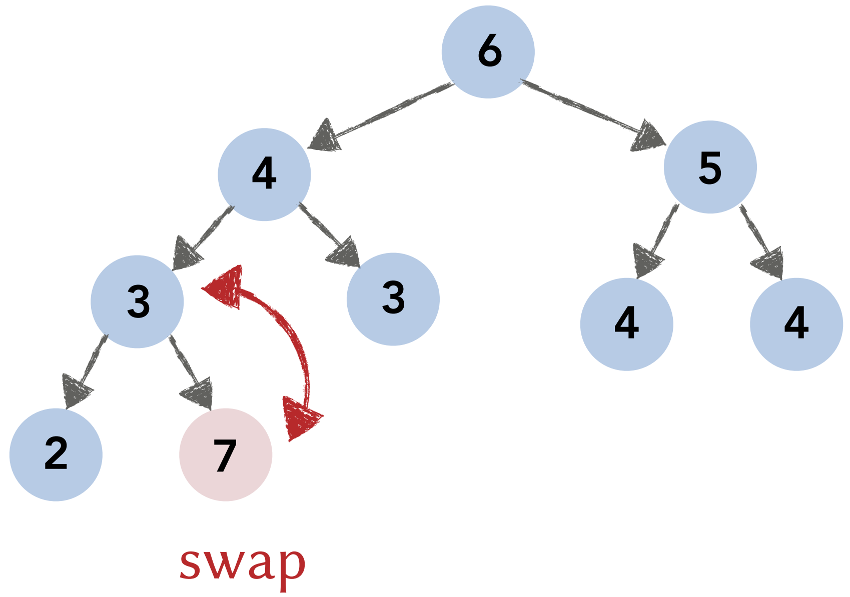
PARENT(i)

```
return (i-1)/2
```

Parent is at index
(3-1)/2 = 1

Fixing a *Locally* Broken Heap

1. If an item becomes **larger** than its parent, push it **up** the tree to maintain the heap order property.



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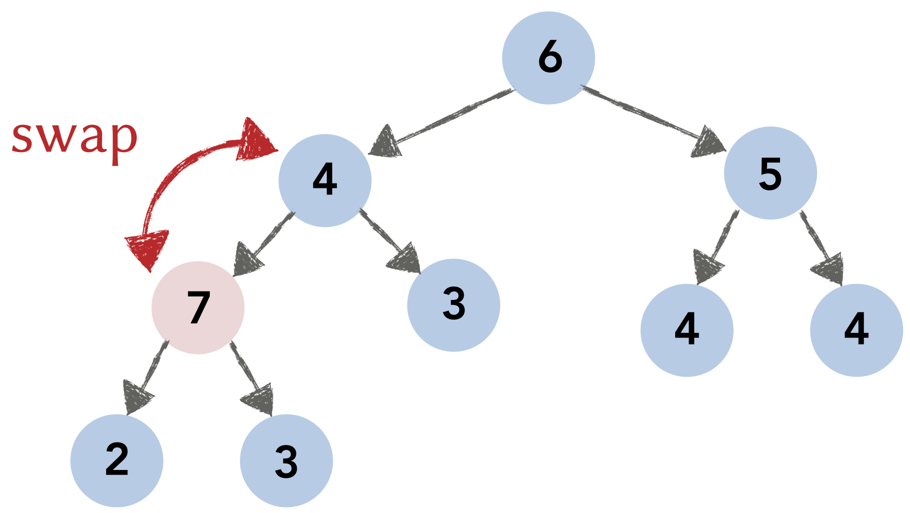

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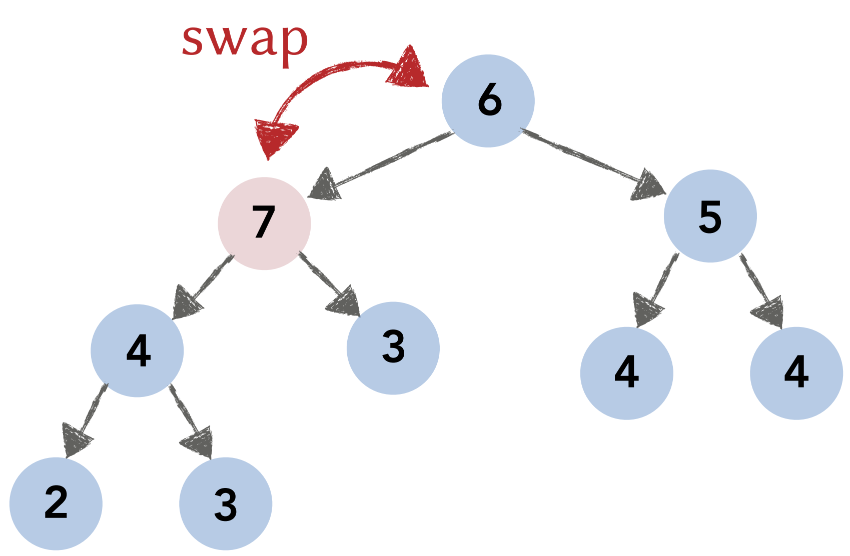

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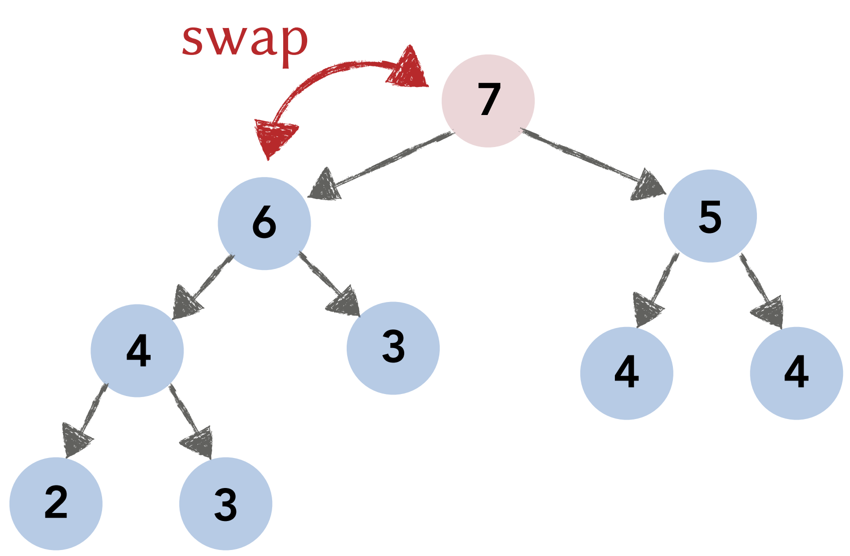

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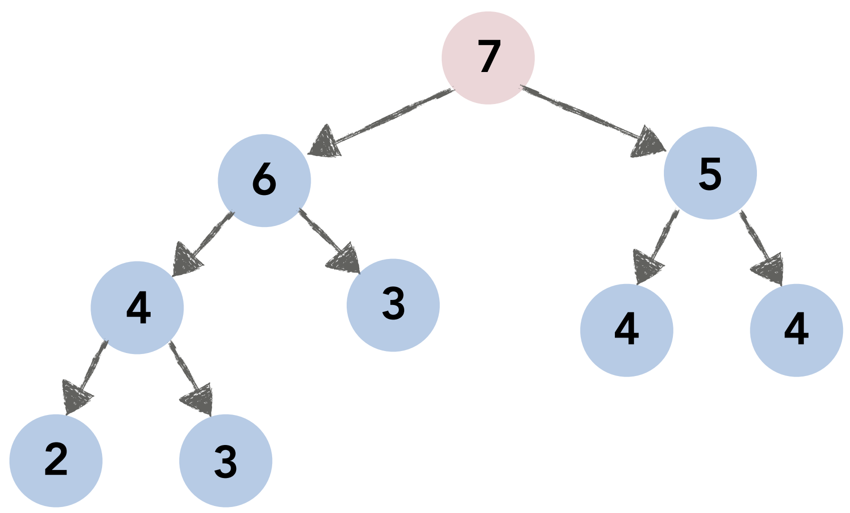

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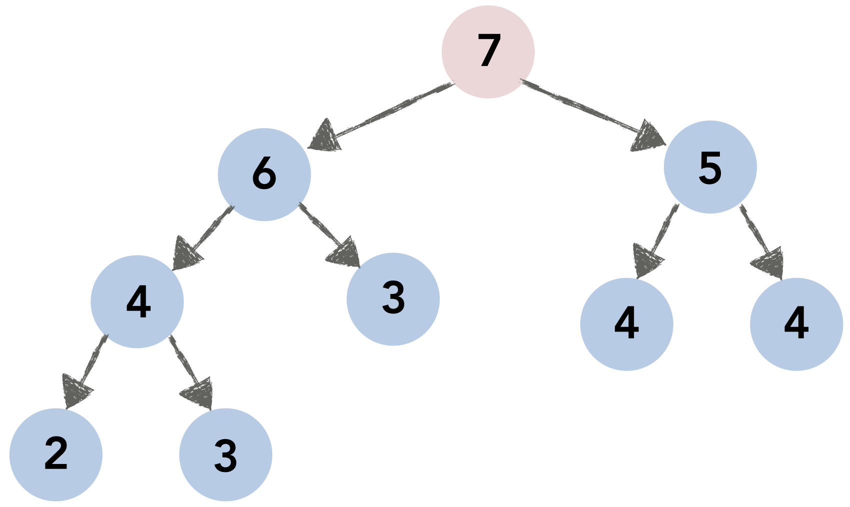

SWIM(a[], i, size)

also called `SiftUp()`
(not `shiftup`) on wikipedia

Fixing a *Locally* Broken Heap

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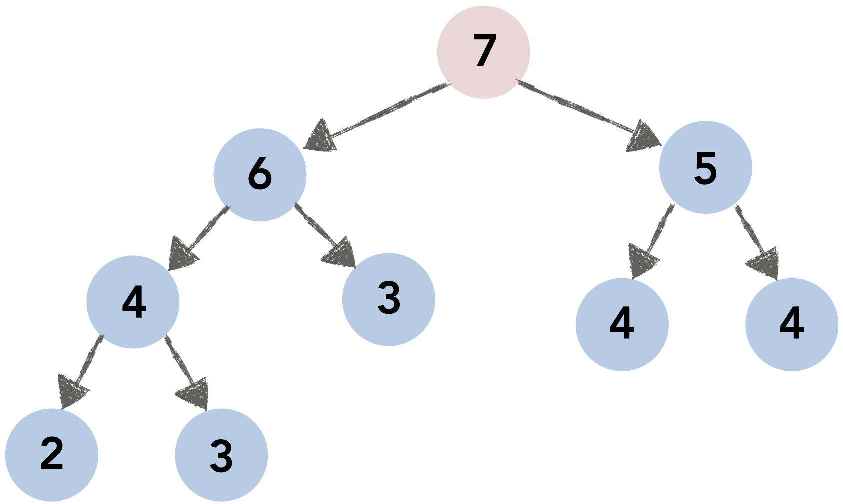
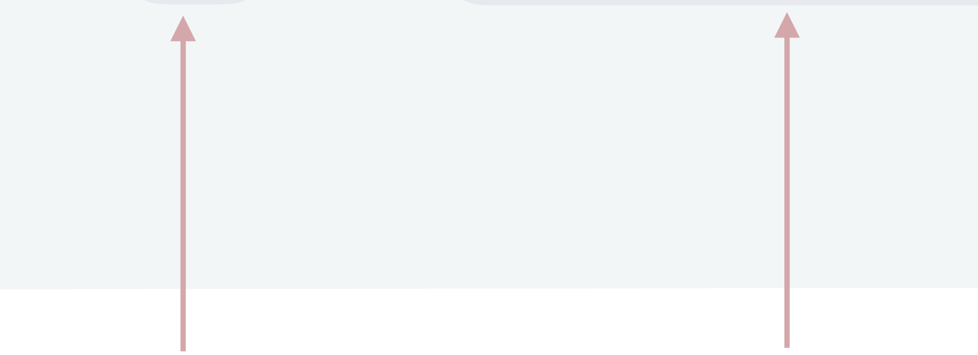



```
SWIM(a[], i, size)

while (i>0 and a[i] > a[PARENT(i)]):
```



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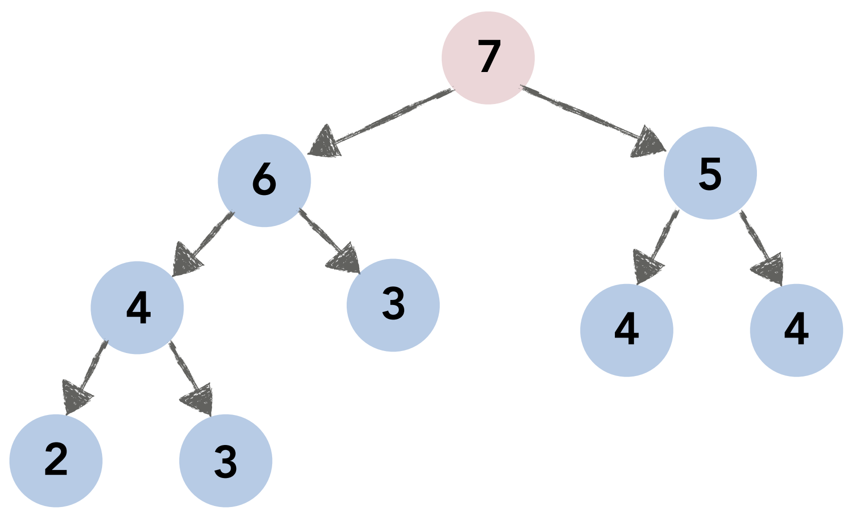


```
SWIM(a[], i, size)

while (i>0 and a[i] > a[PARENT(i)]):
    swap(a[i], a[PARENT(i)])
    i = PARENT(i)
```

swap values with the parent and move to the parent for the next iteration

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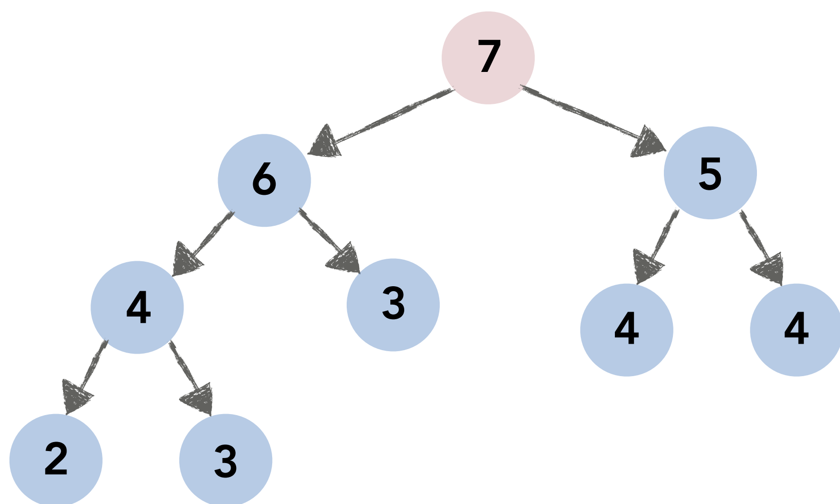


```
SWIM(a[], i, size)

while (i>0 and a[i] > a[PARENT(i)]):
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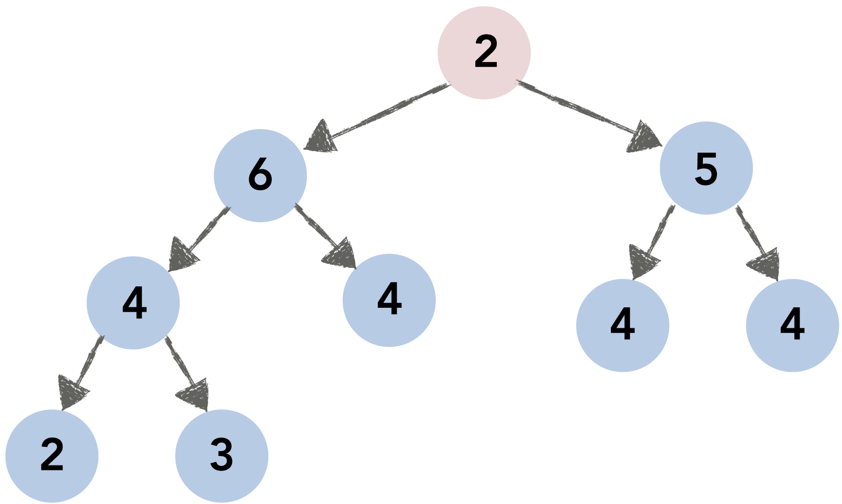
```

Running Time. $O(\log n)$
1 swap and 1 compare per iteration. The number of iterations is bounded by the tree height.

Fixing a *Locally* Broken Heap

2. If an item becomes **less** than one of its children, push it **down** the tree to maintain the heap order property.



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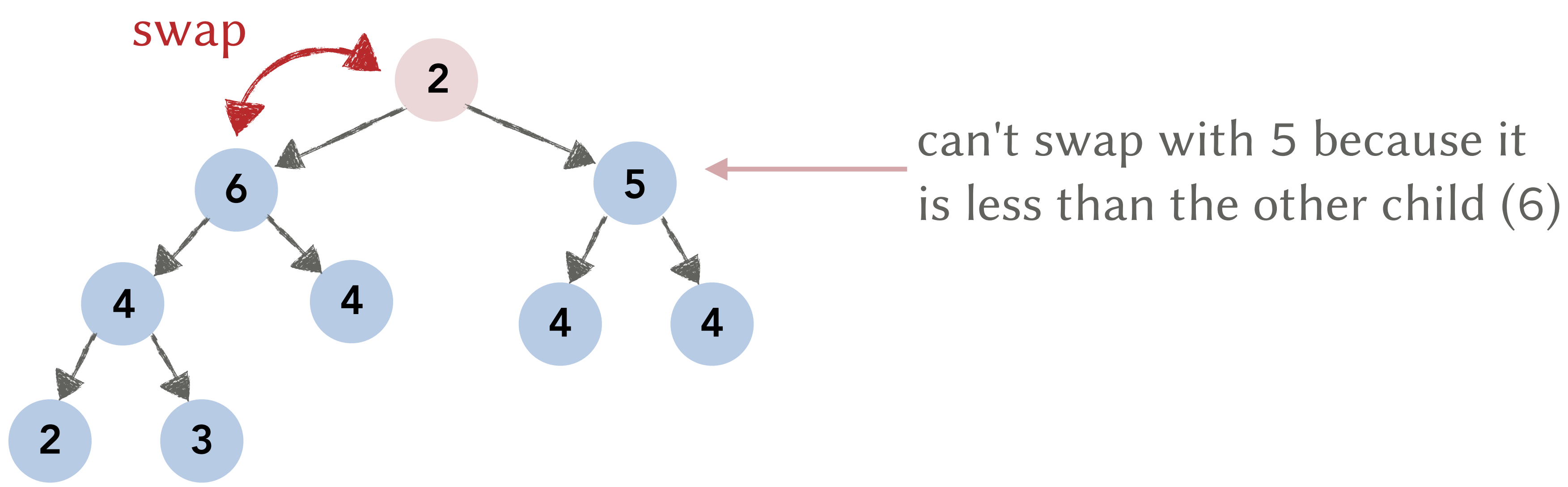

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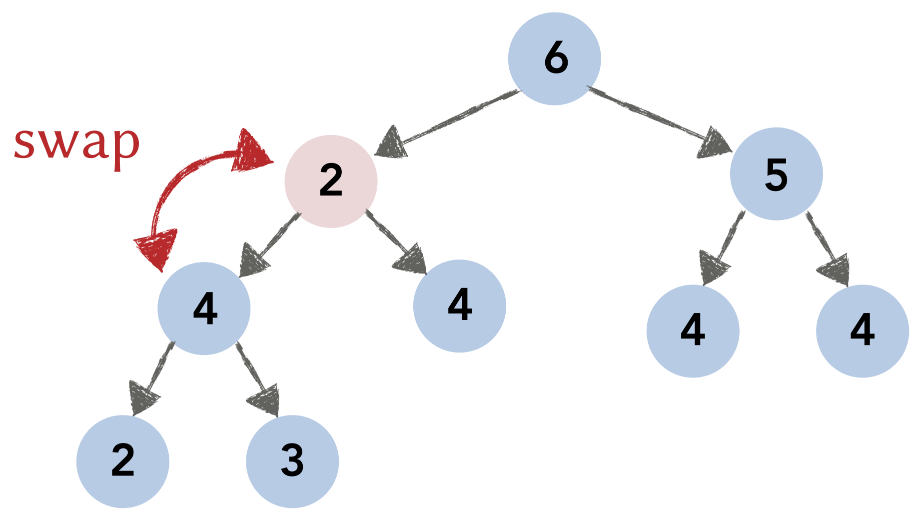

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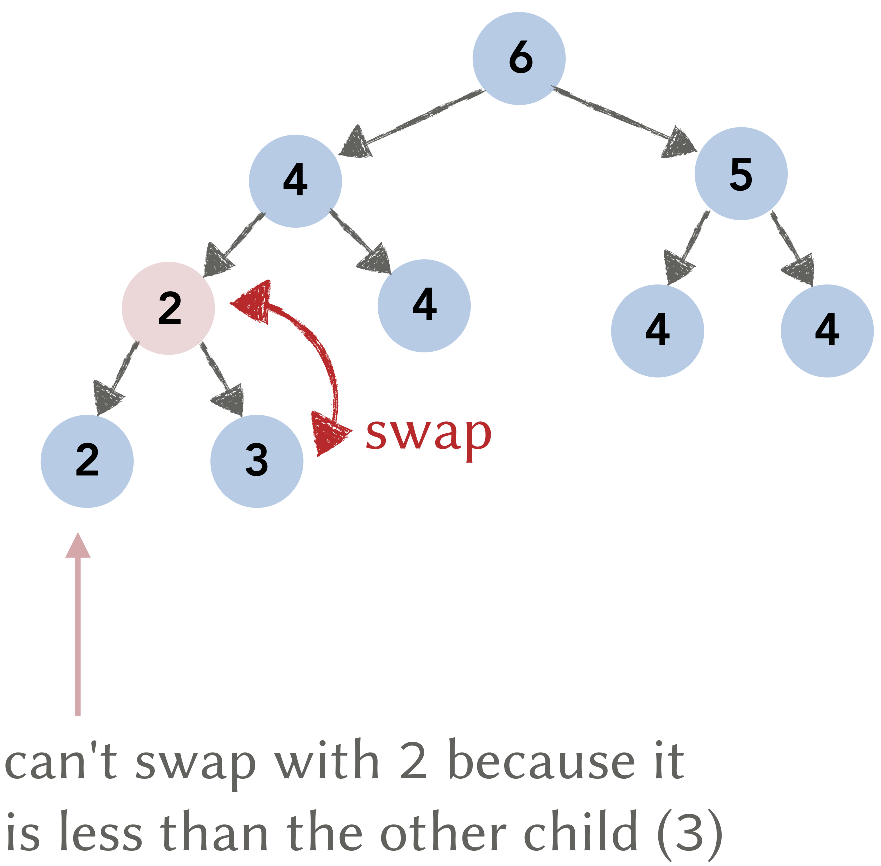

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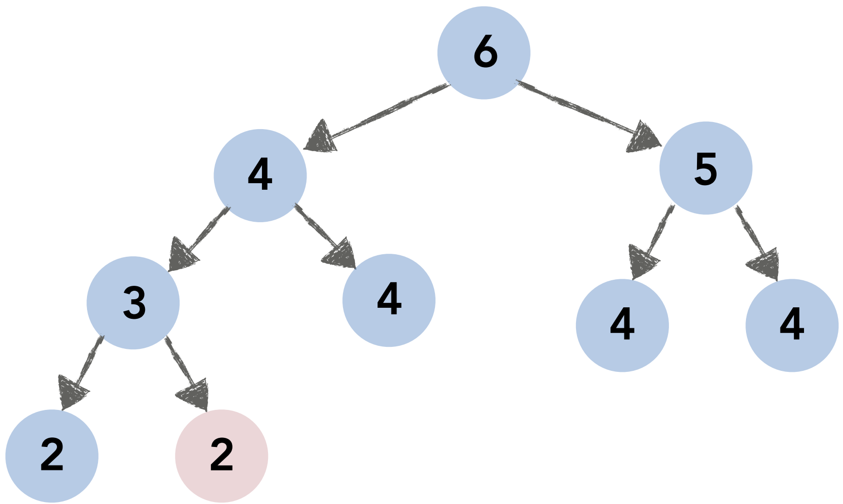

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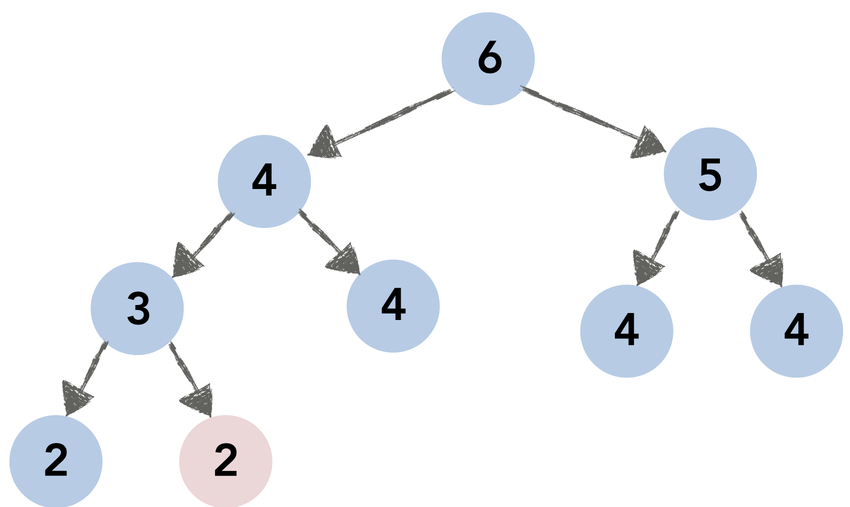


```
SINK(a[], i, size)
```

also called:

- `SIFTDOWN` on wikipedia
- `MAX-HEAPIFY` in our text-book
- `FIX-HEAP` in the slides of the other sections!

Fixing a *Locally* Broken Heap

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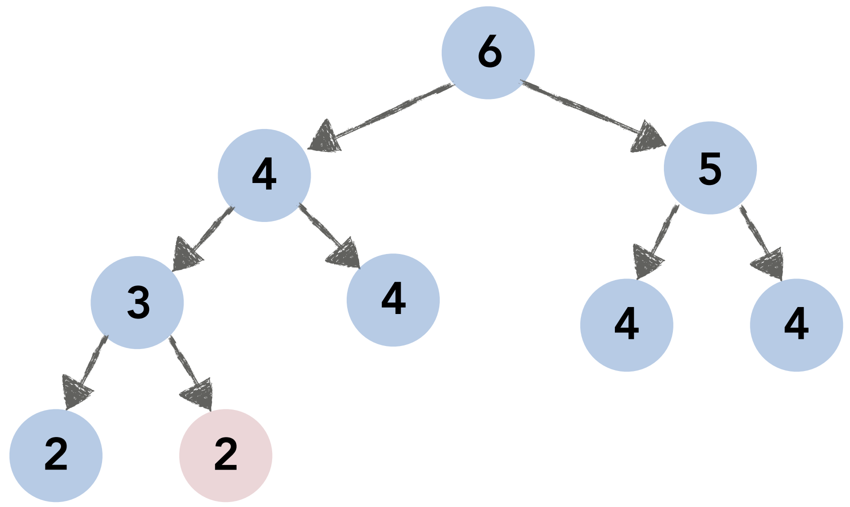


```
SINK(a[], i, size)
while (LEFT(i) < size):
```

while there is a left child

Fixing a *Locally* Broken Heap

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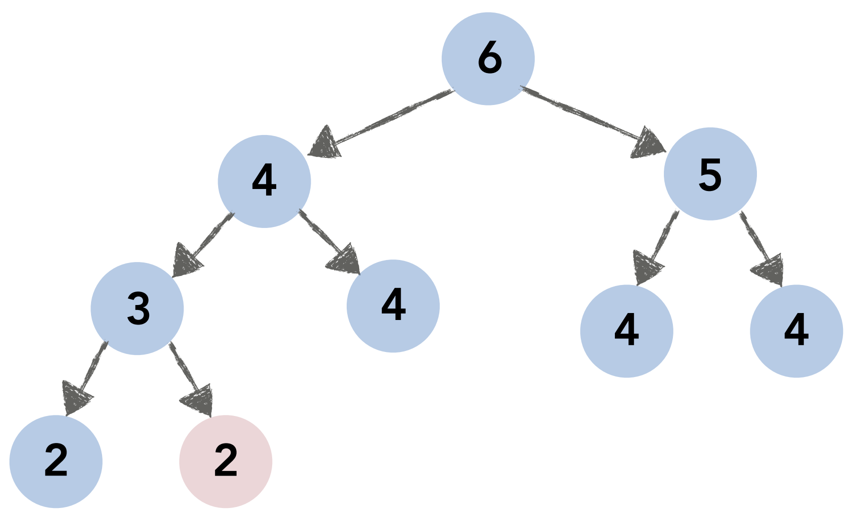


```
SINK(a[], i, size)

while (LEFT(i) < size):
   k = LEFT(i)

   if (RIGHT(i) < size):
      if (a[k] < a[RIGHT(i)]): k = RIGHT(i)
```

pick between the `left` and `right` child depending on which one is the largest.

Fixing a *Locally* Broken Heap

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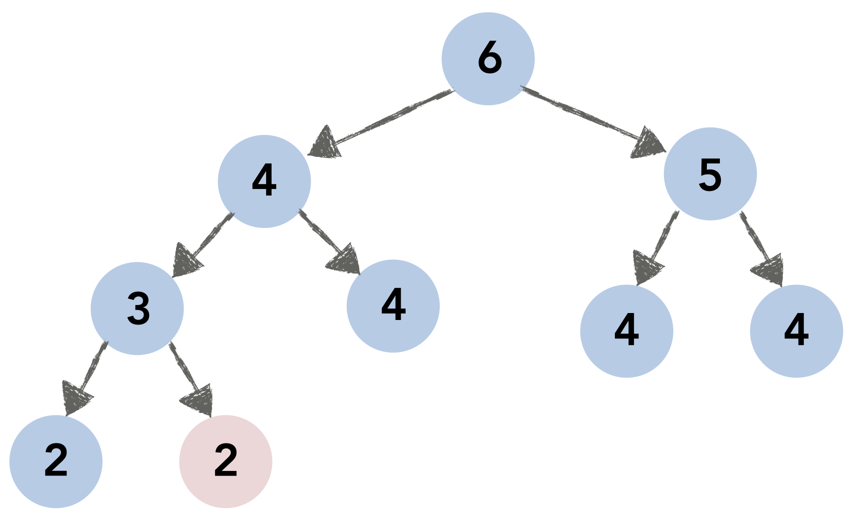


```
SINK(a[], i, size)

while (LEFT(i) < size):
   k = LEFT(i)

   if (RIGHT(i) < size):
      if (a[k] < a[RIGHT(i)]): k = RIGHT(i)
   if (a[i] < a[k]):
      swap(a[i], a[k])
      i = k
   else: break
```

swap with and move to the larger child or stop if no swap is necessary

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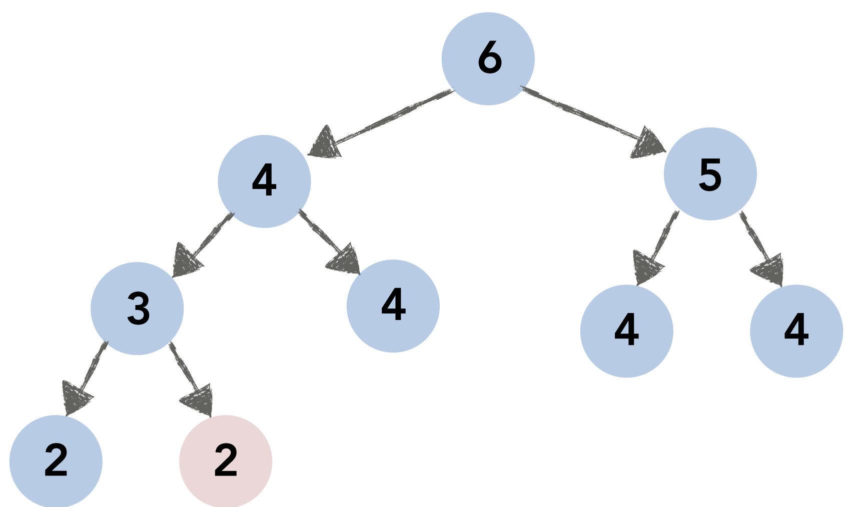


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   if (a[i] < a[k]):
      swap(a[i], a[k])
      i = k
   else: break
```

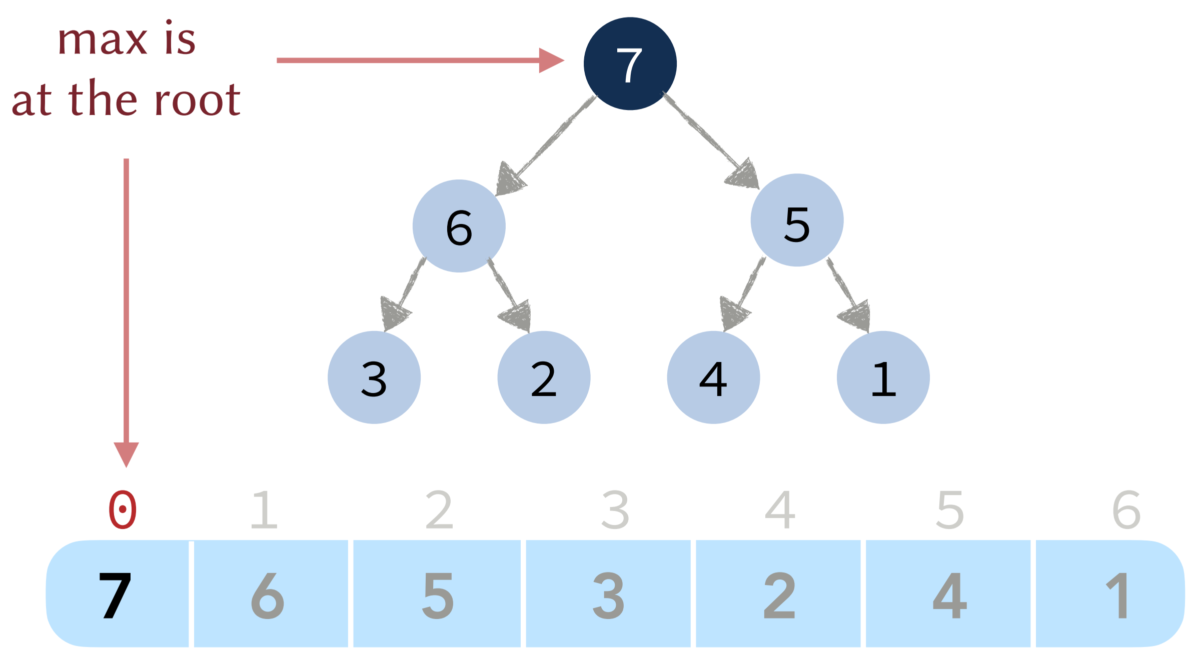
Running Time. $O(\log n)$

At most 1 swap and 2 comparisons per iteration

The number of iterations is bounded by the tree height.

Max-PQ Operations

Max: Always at index 0.
$\Theta(1)$



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Insert: Insert at the end of the array and then **swim.**

INSERT(a[], k, size)

```
a[size] = k
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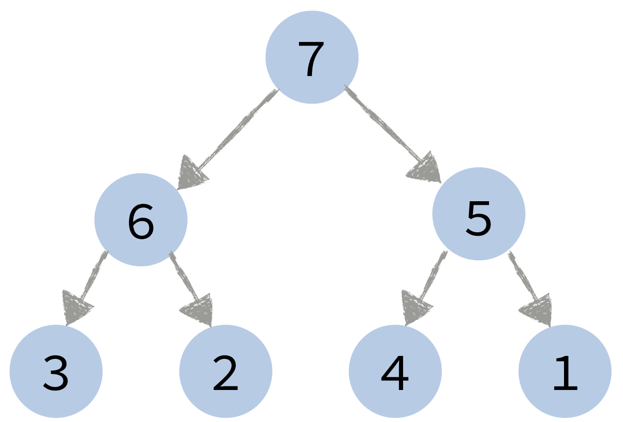
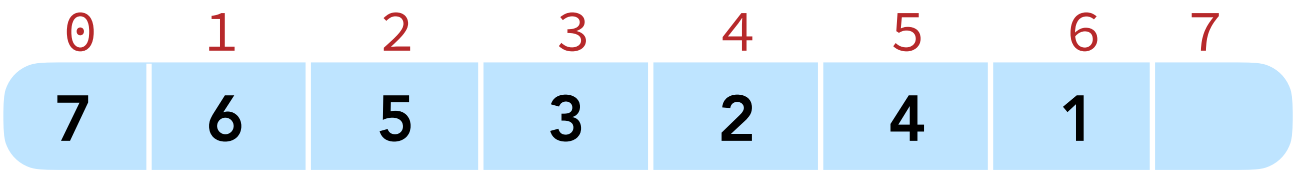
SWIM(a, size-1, size)
```

$O(\log n)$: Swim at most to the root.





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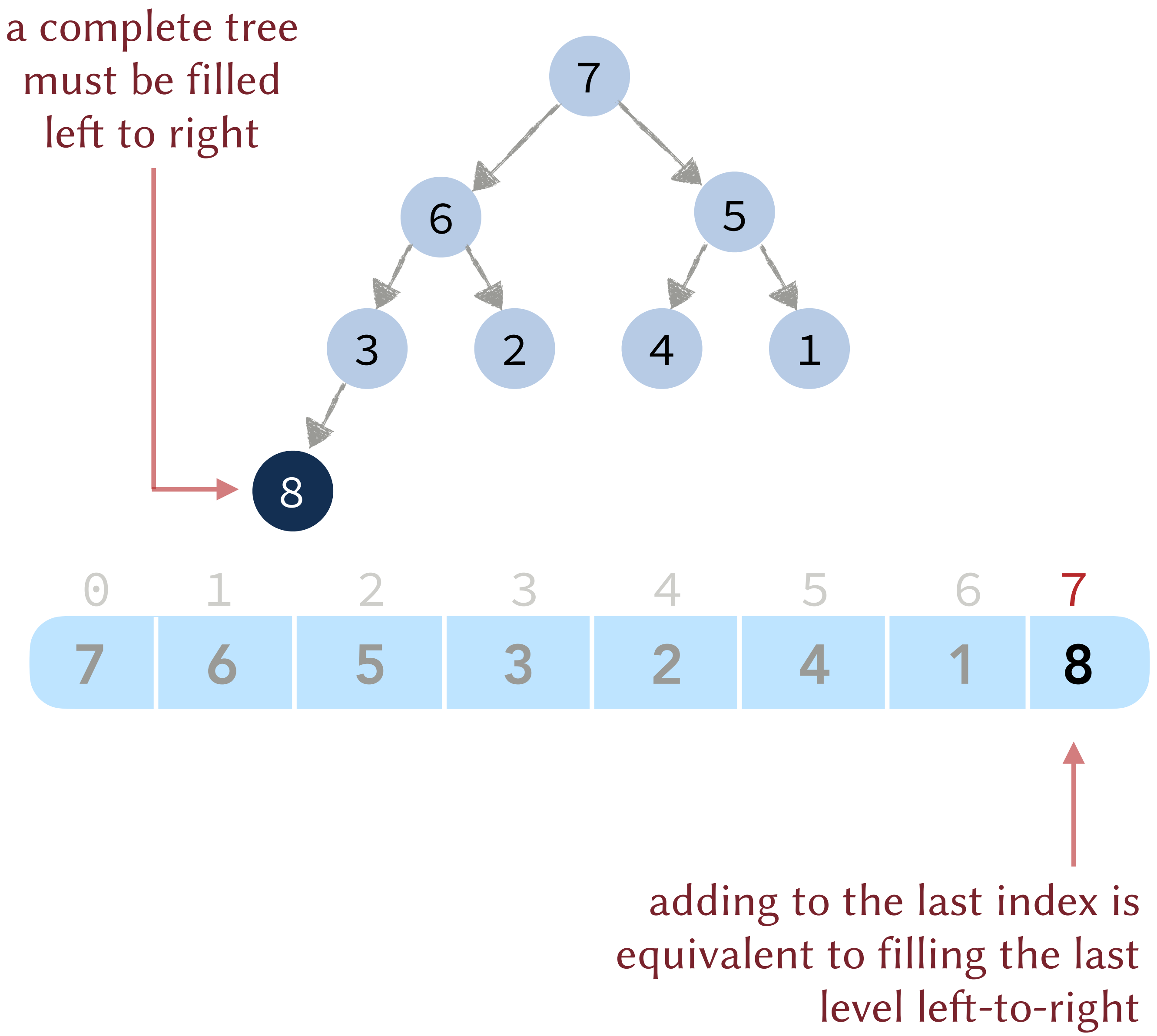

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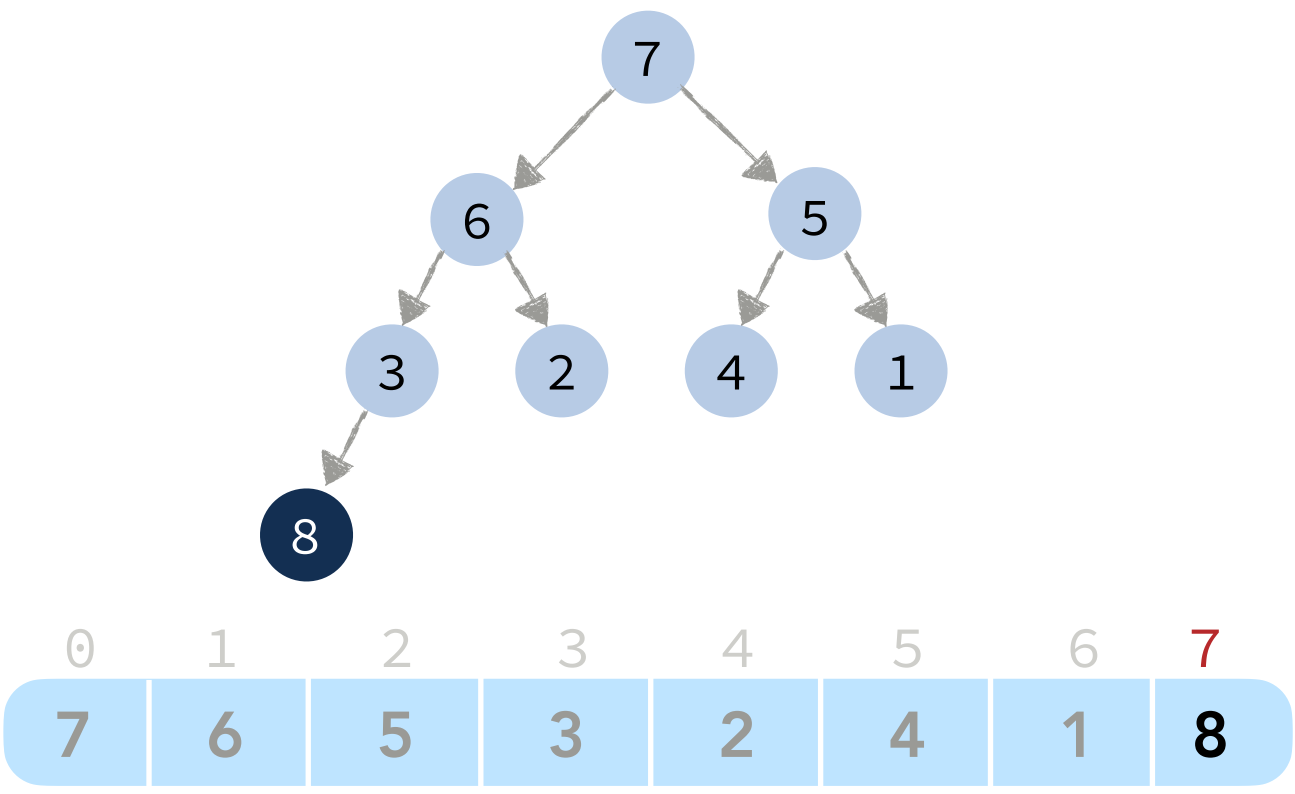
fix the heap after inserting the new element



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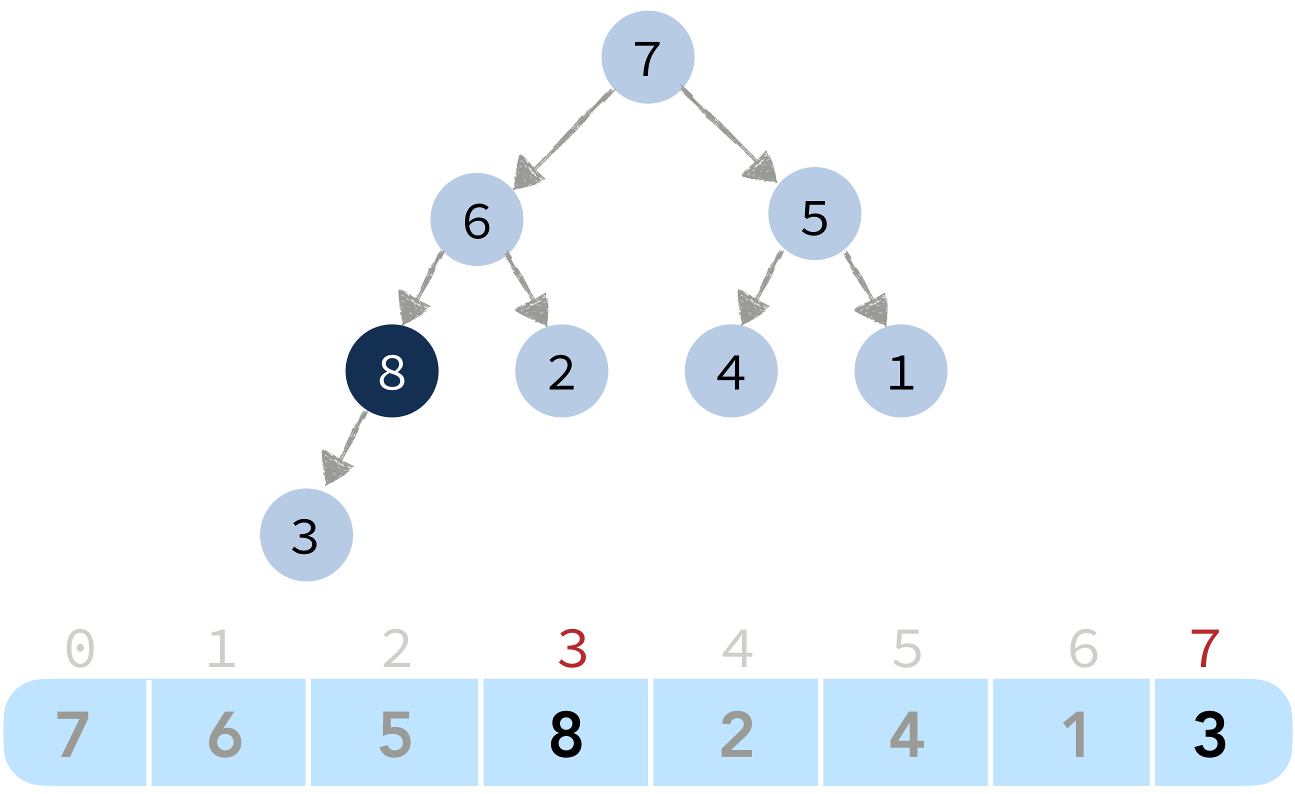

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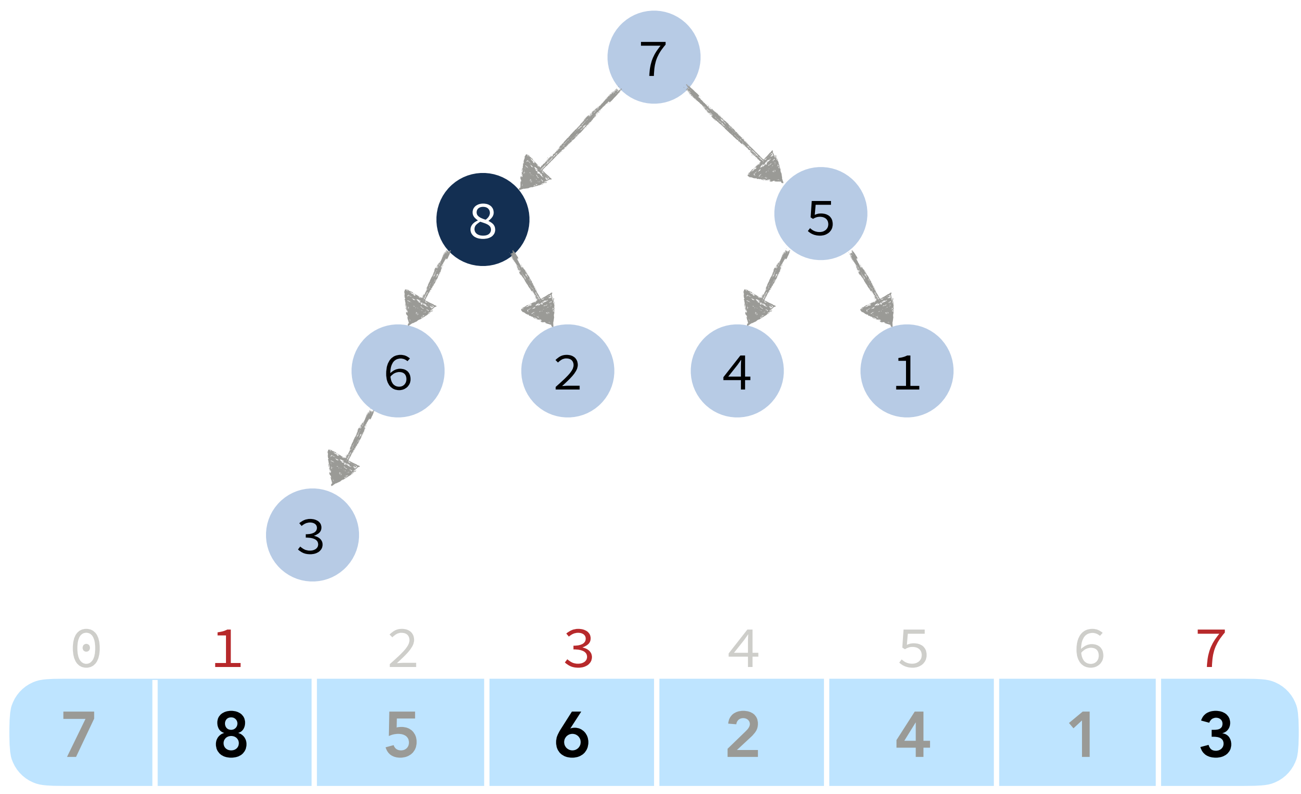

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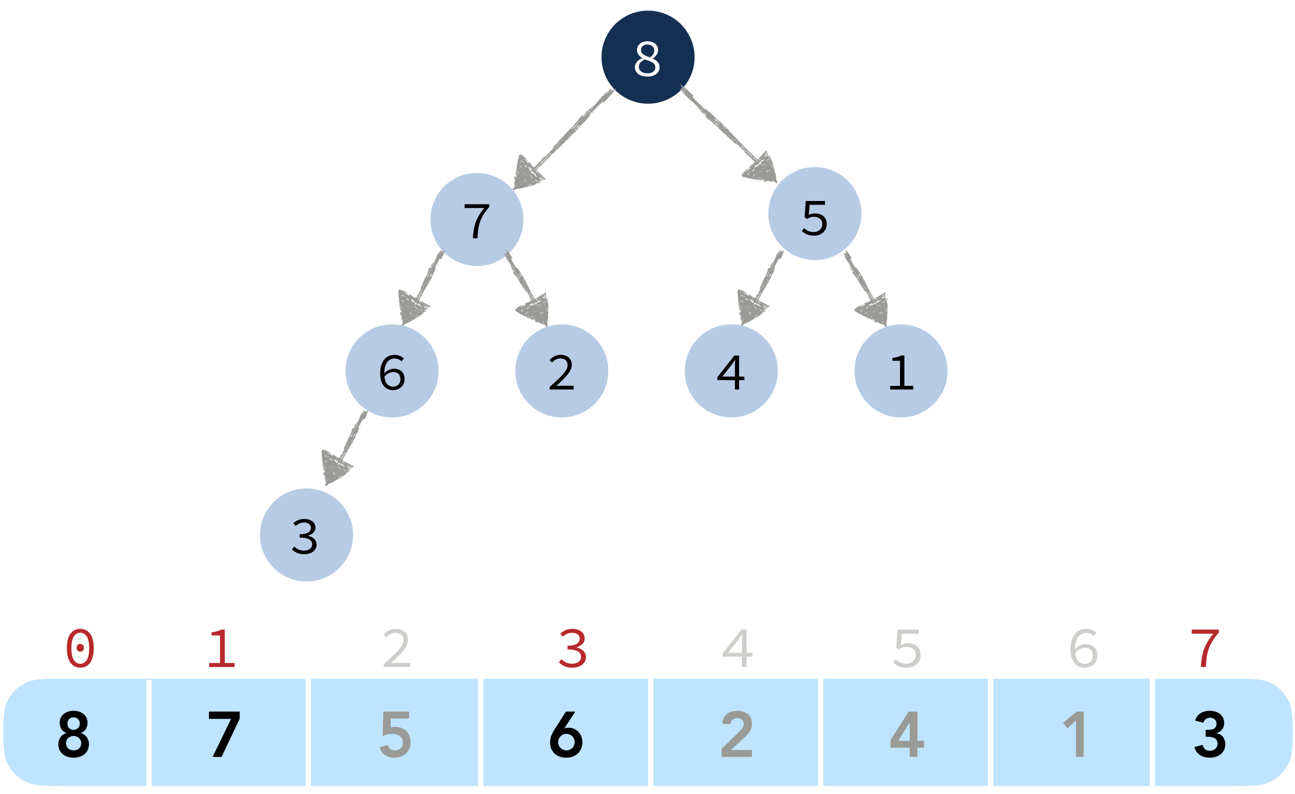

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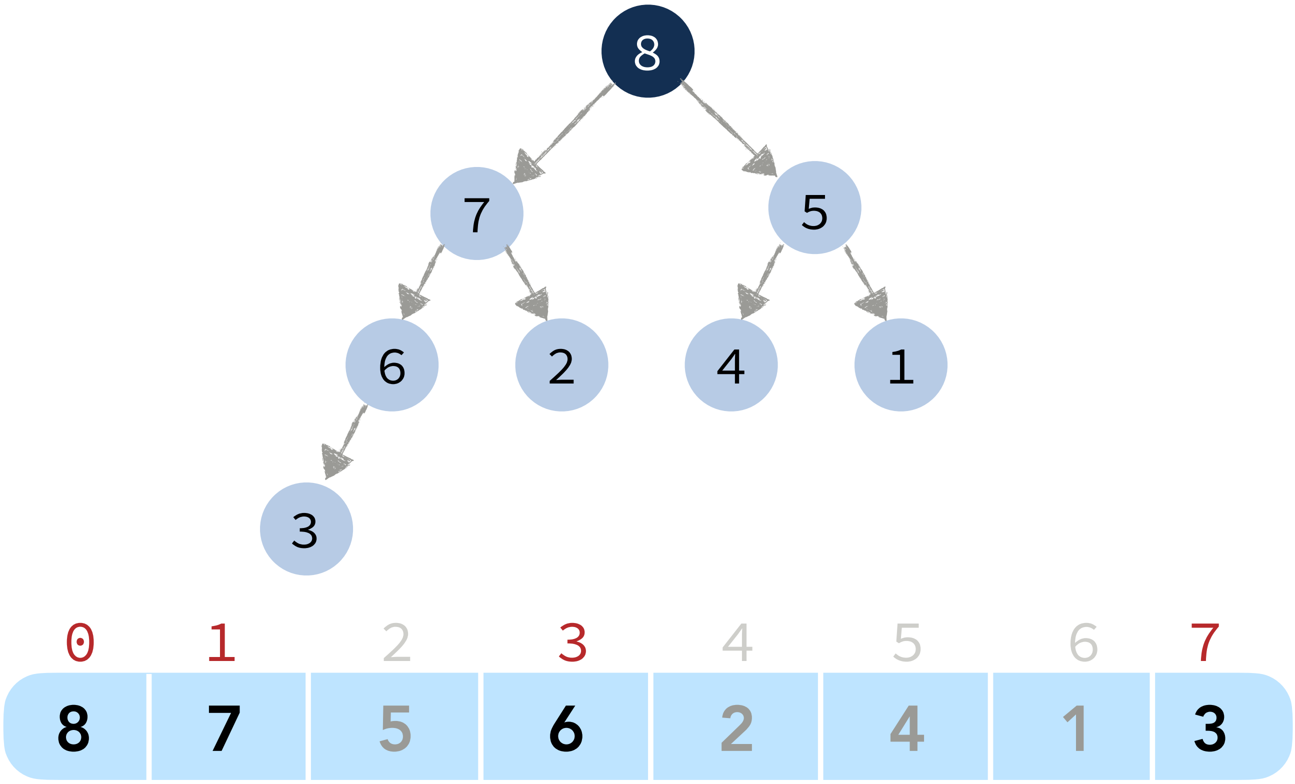

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```
DEL-MAX(a[], size)

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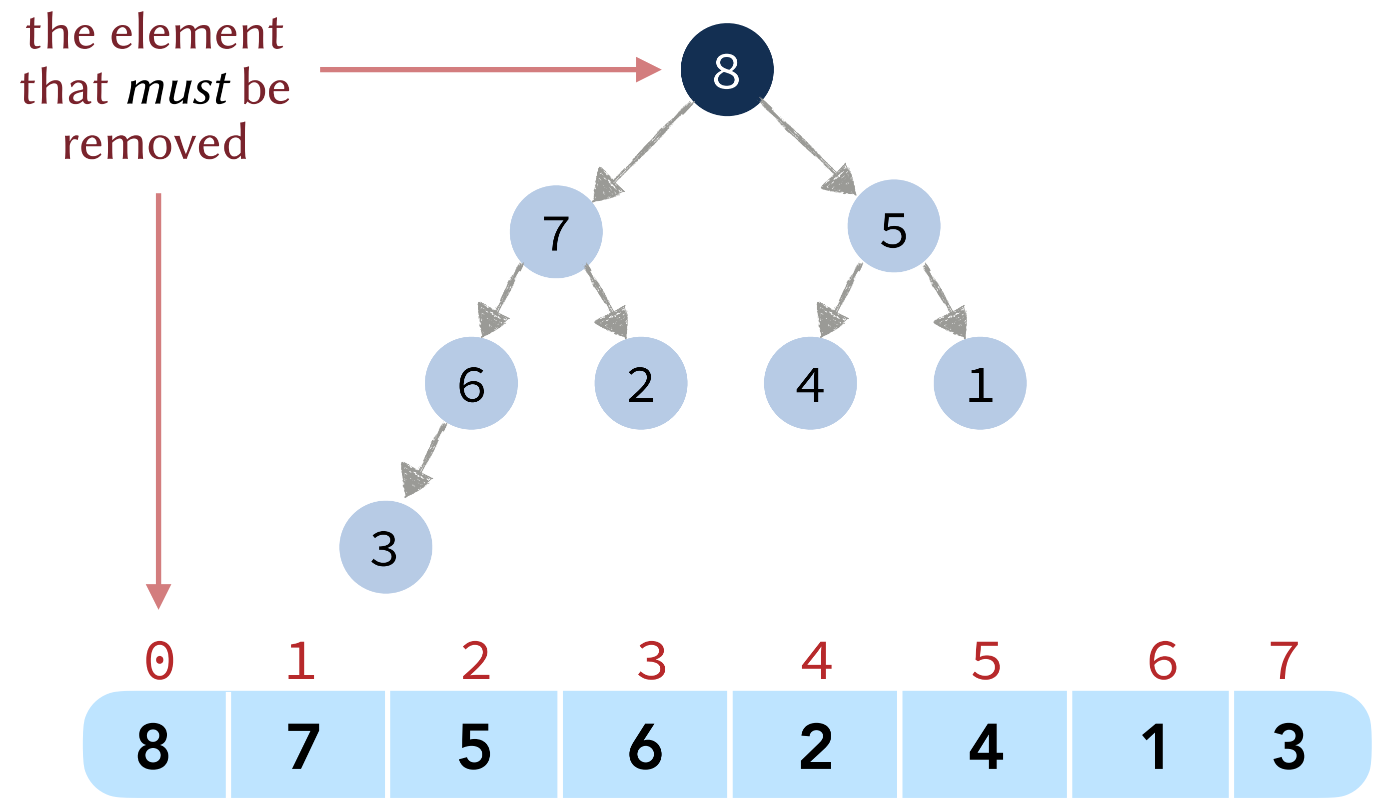

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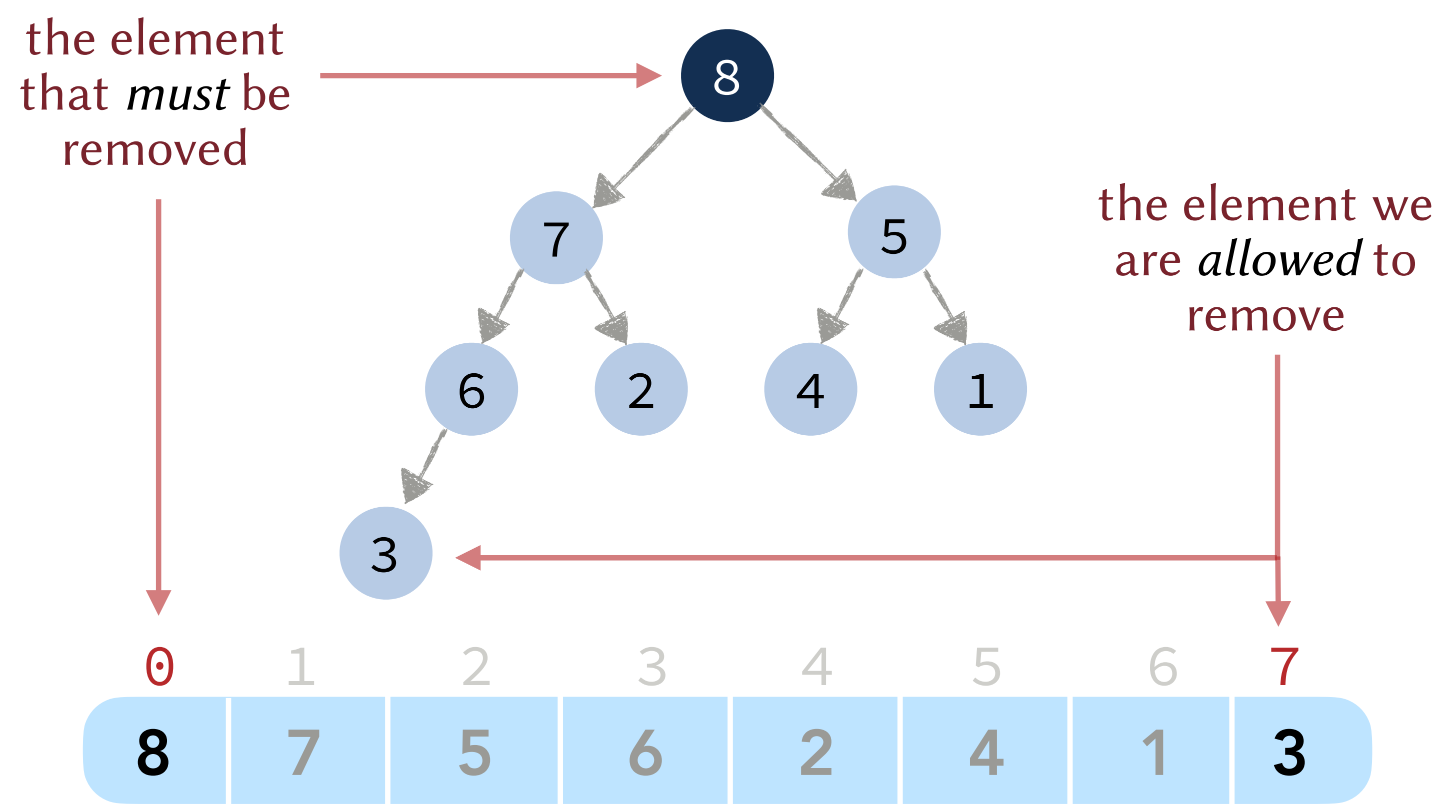

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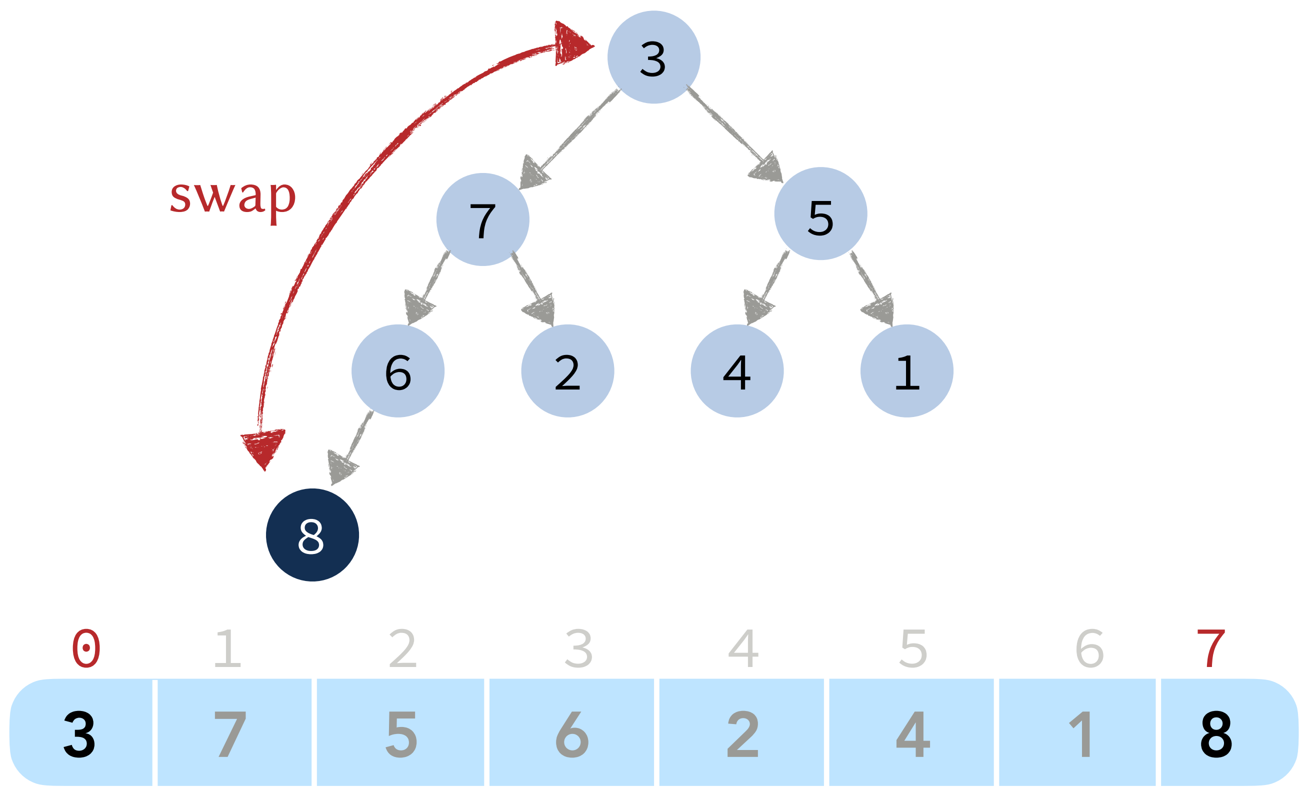

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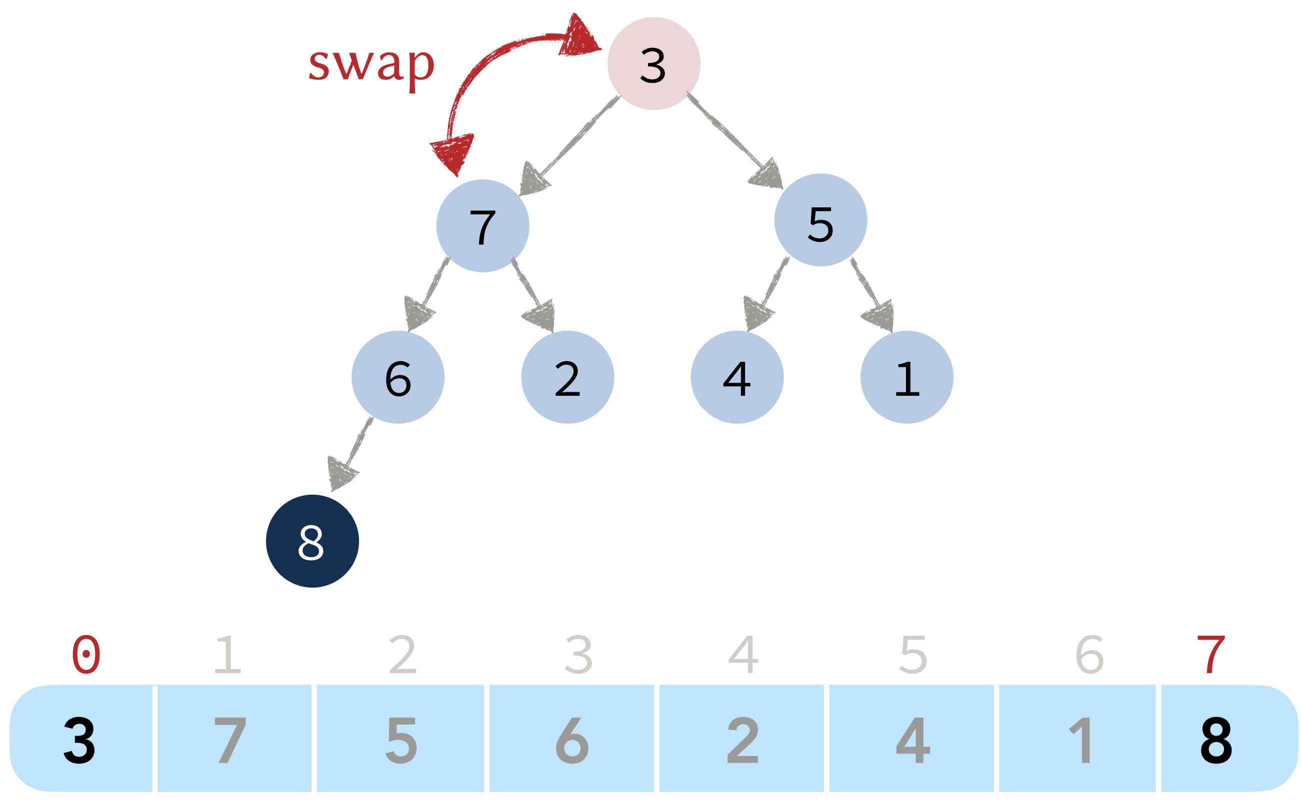

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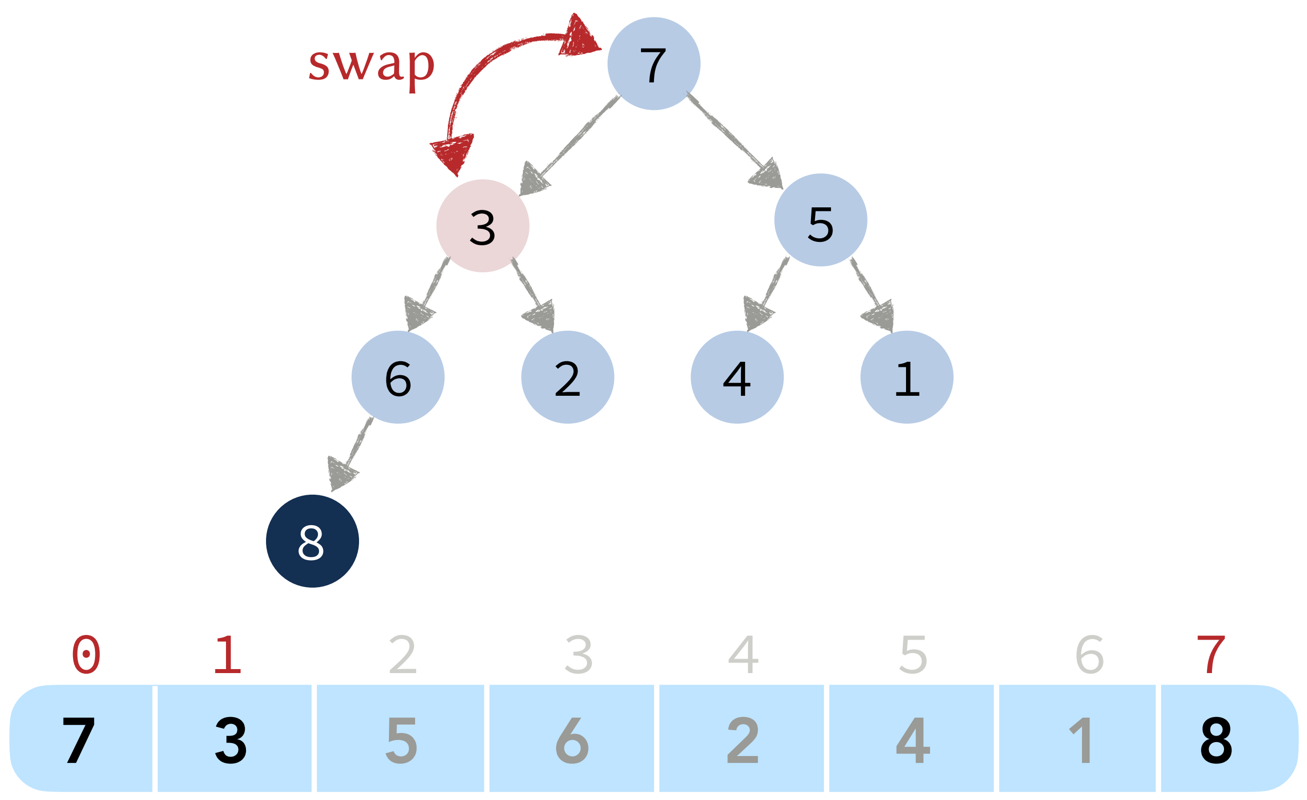

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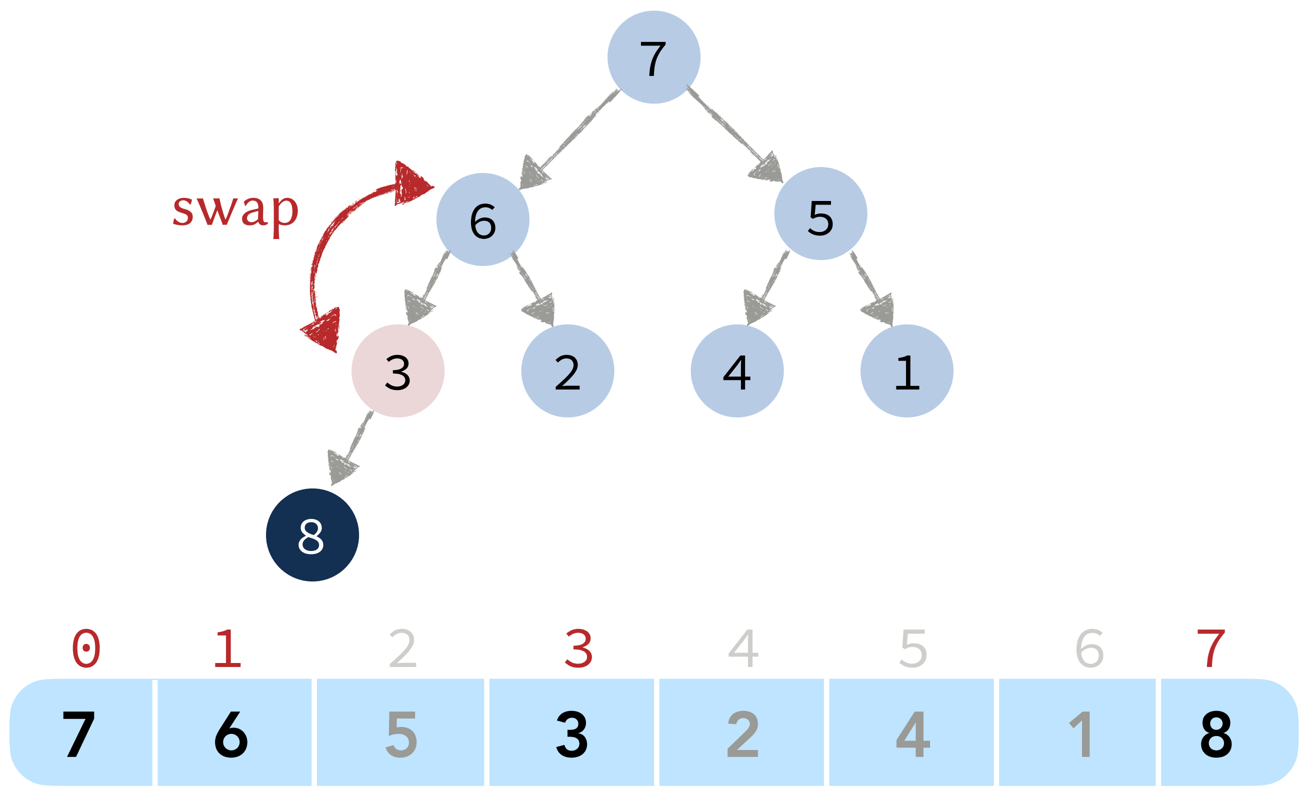

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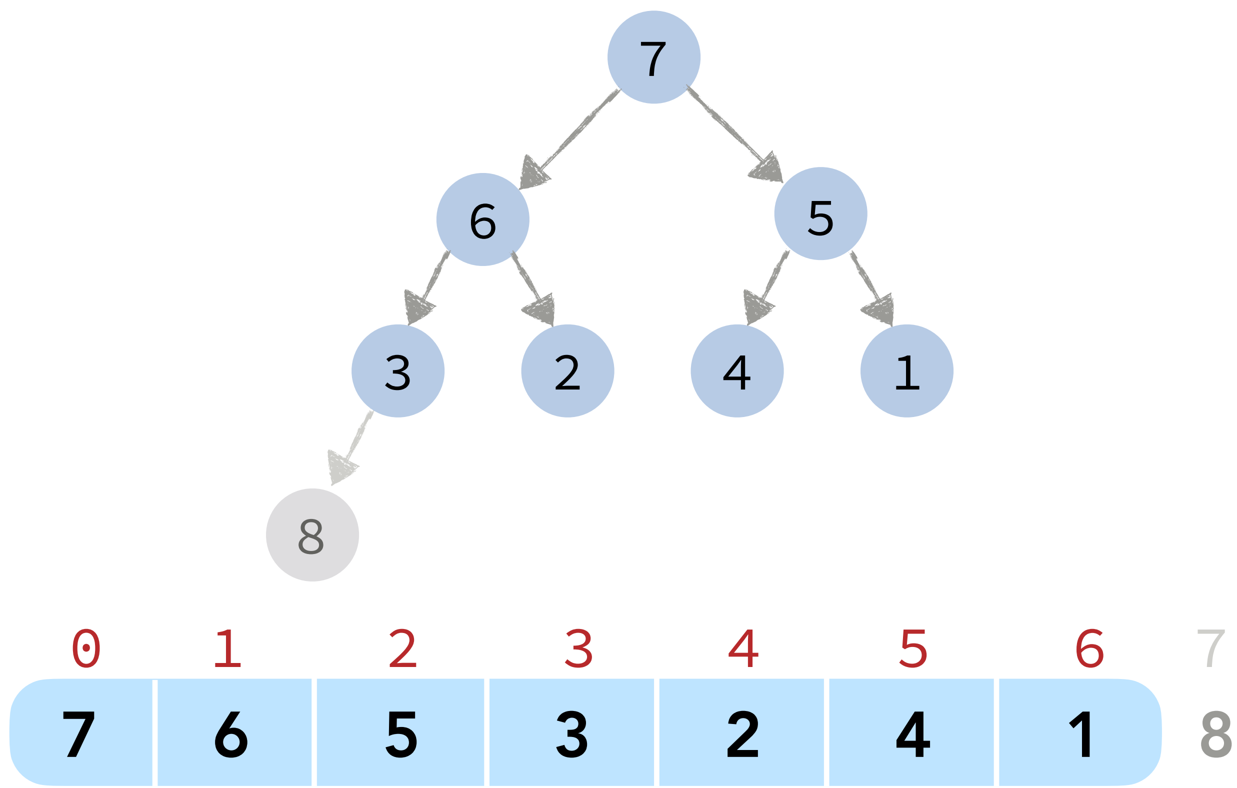

Quiz # 1

Implement a max-PQ that supports the following operation:

`del-Random`: Removes a random element from the priority queue.

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Answer.

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DEL-RANDOM(a[], size)

 k = random index in [0, size-1]
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 size = size-1
 SINK(a, k, size)
 SWIM(a, k, size)
```

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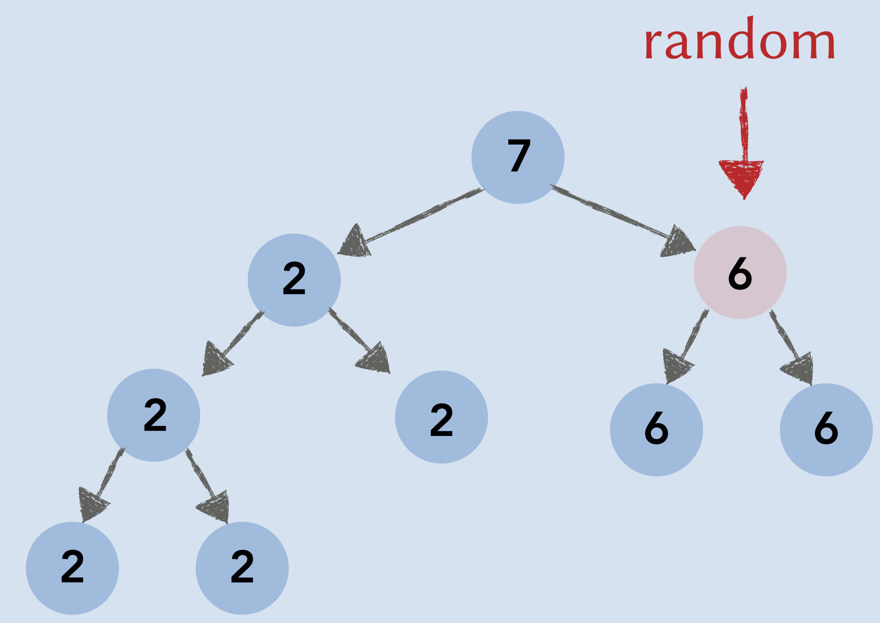

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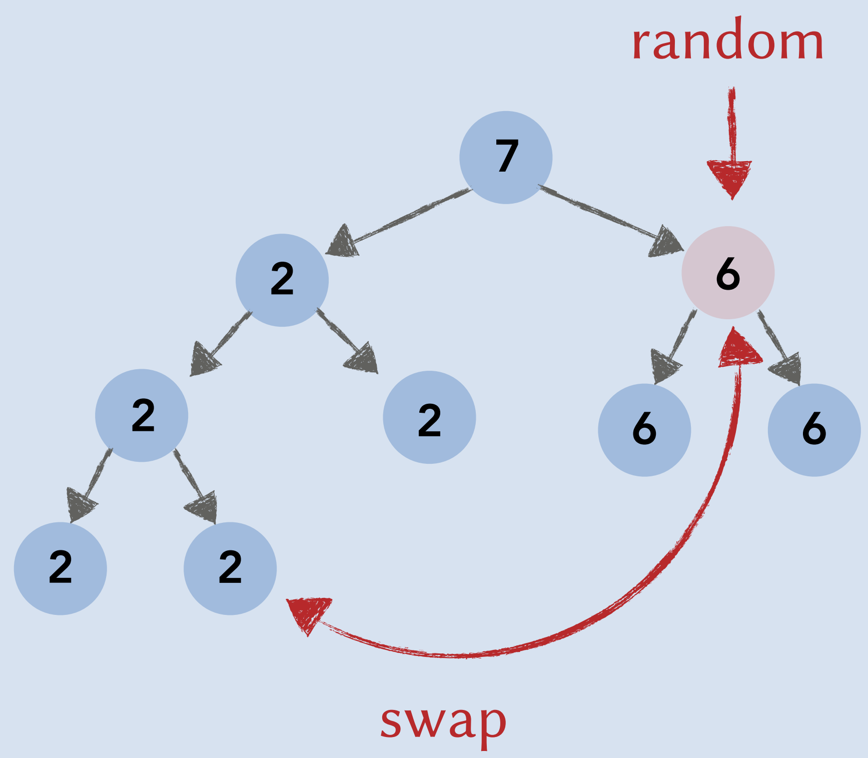

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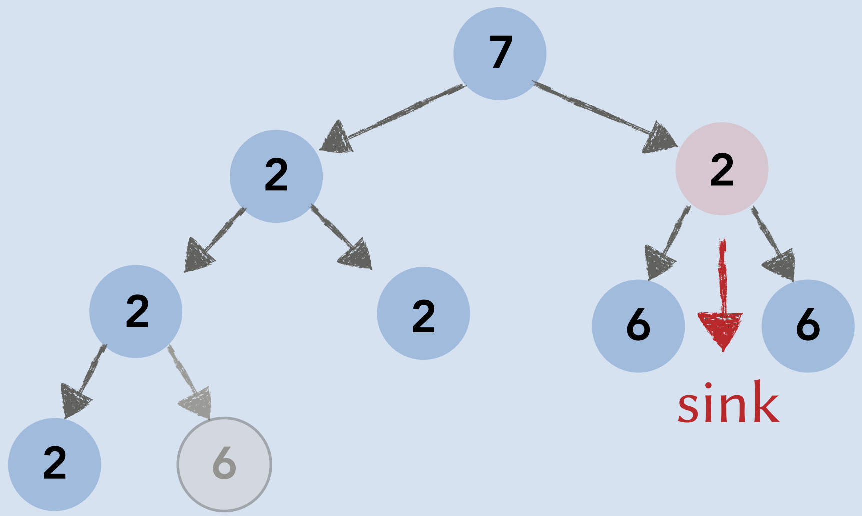

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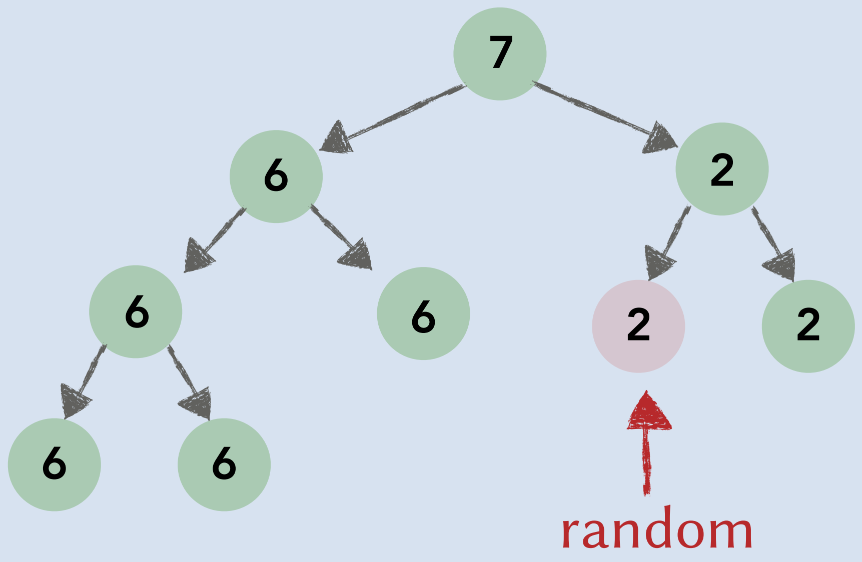

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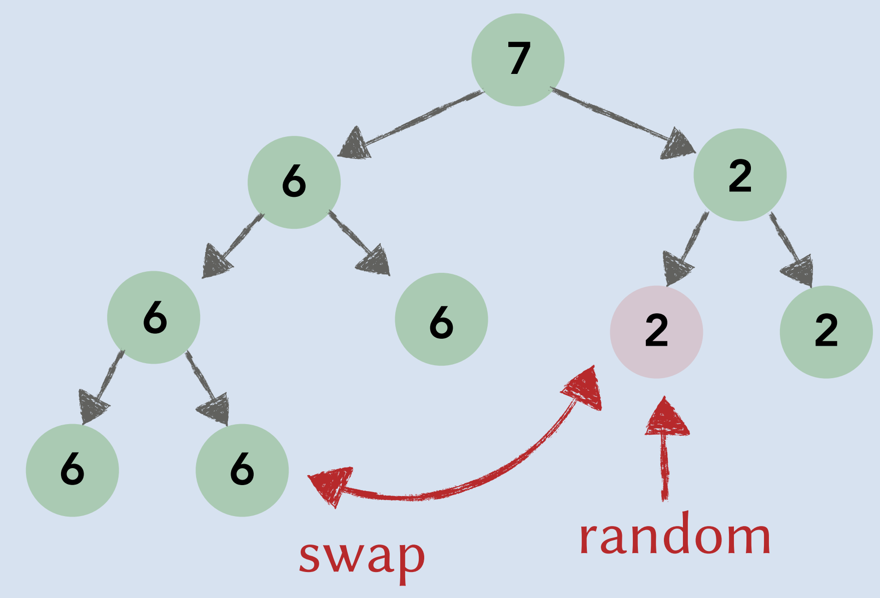

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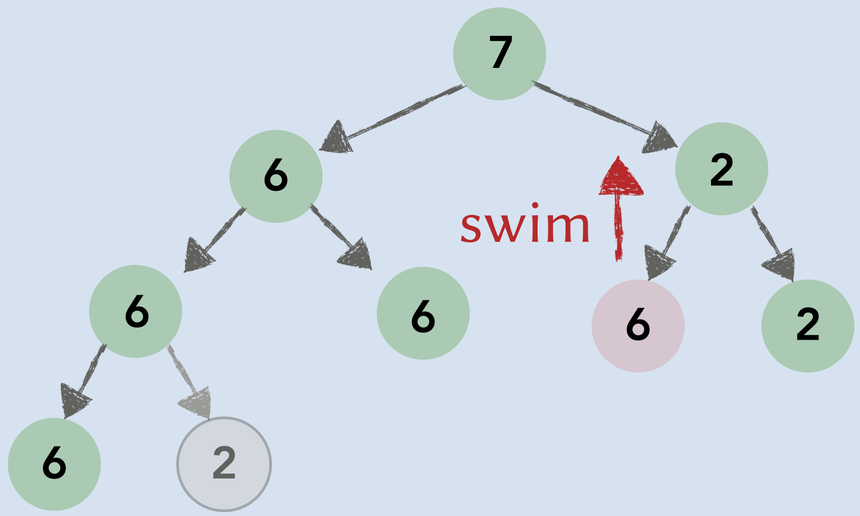

Heapsort

Heapsort: Naive Implementation

HEAP-SORT(a[], size)

```
heap ← An empty max heap
for i = 0 → n-1:
    heap.INSERT(a[i])

for i = n-1 → 0:
    a[i] = heap.MAX()
    heap.DELETE-MAX()
```

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1 insert all the array elements into a max-heap

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    heap.INSERT(a[i])

for i = n-1 → 0:
    a[i] = heap.MAX()
    heap.DELETE-MAX()
```

1. insert all the array elements into a max-heap
2. copy all the elements back from the heap to the array (in order)

Heapsort: Naive Implementation

```
HEAP-SORT(a[], size)

heap ← An empty max heap
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Running Time. (number of compares in the worst case)

- Step 1. $\log_2(1) + \log_2(2) + \log_2(3) + \ \ldots \ + \log_2(n-1) \leq \log_2(n!)$

Heapsort: Naive Implementation

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- Step 1. $\log_2(1) + \log_2(2) + \log_2(3) + \ \dots\ + \log_2(n-1) \leq \log_2(n!)$

insert the second element into a heap of size 1

insert the last element into a heap of size n -1

Heapsort: Naive Implementation

```
HEAP-SORT(a[], size)

heap ← An empty max heap
for i = 0 → n-1:
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Running Time. (number of compares in the worst case)

- Step 1. $\log_2(1) + \log_2(2) + \log_2(3) + \ \ldots \ + \log_2(n-1) \le \log_2(n!) = O(n \log n)$
- Step 2. $2 \times (\log_2(n-1) + \log_2(n-2) + \log_2(n-3) + \ \ldots \ + \log_2(1))$
  $\le 2 \times \log_2(n!)$

Heapsort: Naive Implementation

```
HEAP-SORT(a[], size)

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  $\le 2 \times \log_2(n!) = O(n \log n)$

check the analysis of the **SINK** operation!

Heapsort: Naive Implementation

```
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  $\leq 2 \times \log_2(n!) = O(n \log n)$
- Total. $O(n \log n)$

Heapsort: Naive Implementation

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- Step 2. $2 \times (\log_2(n-1) + \log_2(n-2) + \log_2(n-3) + \ \dots \ + \log_2(1))$
  $\leq 2 \times \log_2(n!) = O(n \log n)$
- Total. $O(n \log n)$

🤔 **Can we do better?**

Not asymptotically, but we can still improve the actual running time!

Heapsort: A Better Implementation

```
HEAP-SORT(a[], size)

  CONSTRUCT-HEAP(a, size)

  while (size > 1):
      swap(a[0], a[size-1])
      size = size-1
      SINK(a, 0, size)
```

0	1	2	3	4	5	6	7	8
6	7	6	2	4	4	5	2	5

random array

size-1

Heapsort: A Better Implementation

```
HEAP-SORT(a[], size)

    CONSTRUCT-HEAP(a, size)

    while (size > 1):
        swap(a[0], a[size-1])
        size = size-1
        SINK(a, 0, size)
```

1 construct a max-heap **in-place** (**convert** the array to become a heap)

How? stay tuned!

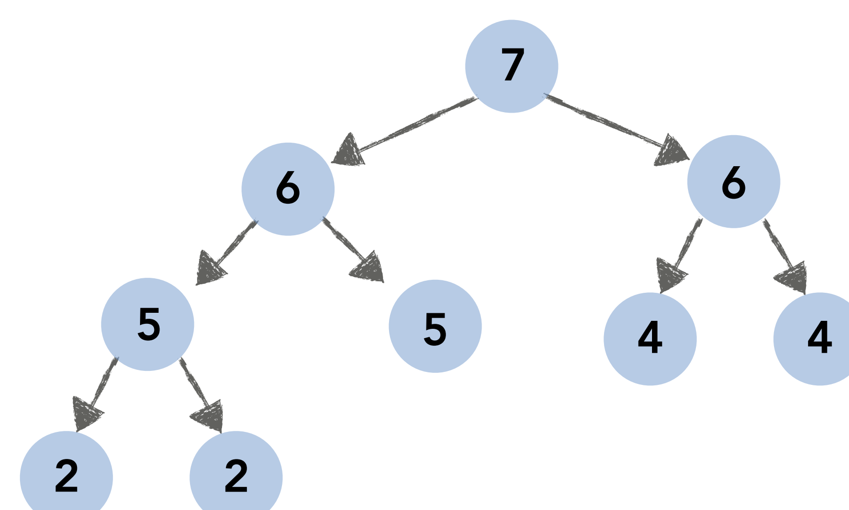
0	1	2	3	4	5	6	7	8
6	7	6	2	4	4	5	2	5

random array — size-1

CONSTRUCT-HEAP()

0	1	2	3	4	5	6	7	8
7	6	6	5	5	4	4	2	2

max-heap — size-1



Heapsort: A Better Implementation

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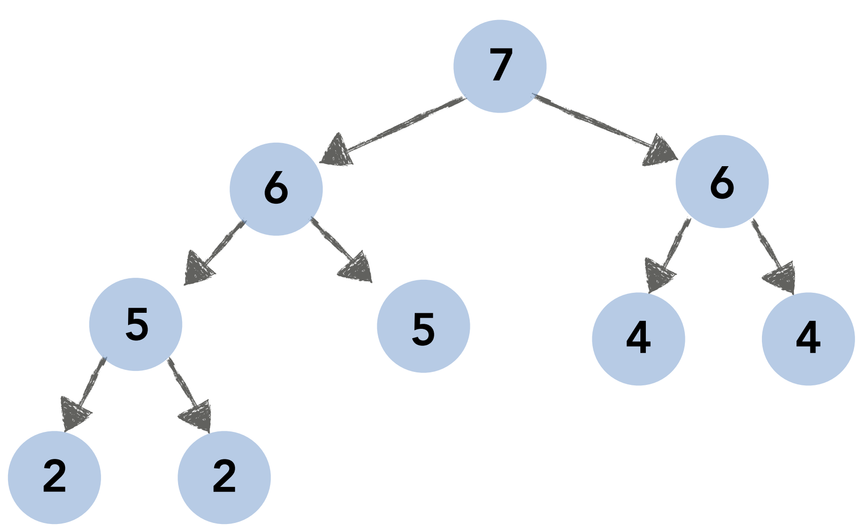
1. construct a max-heap in-place (change the array to become a heap)
2. repeatedly place the next maximum in its right position

0	1	2	3	4	5	6	7	8
7	6	6	5	5	4	4	2	2

max-heap

size-1



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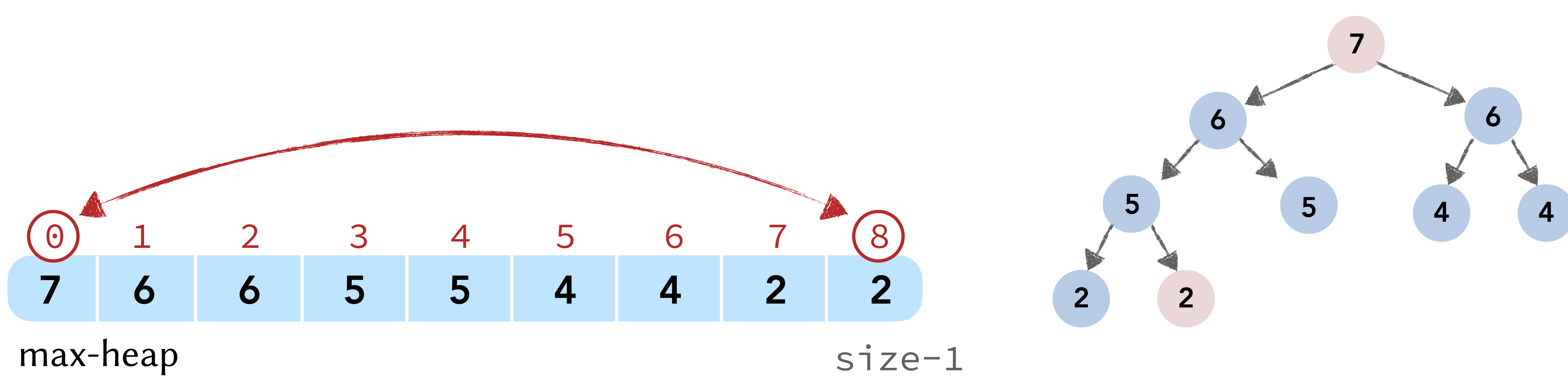

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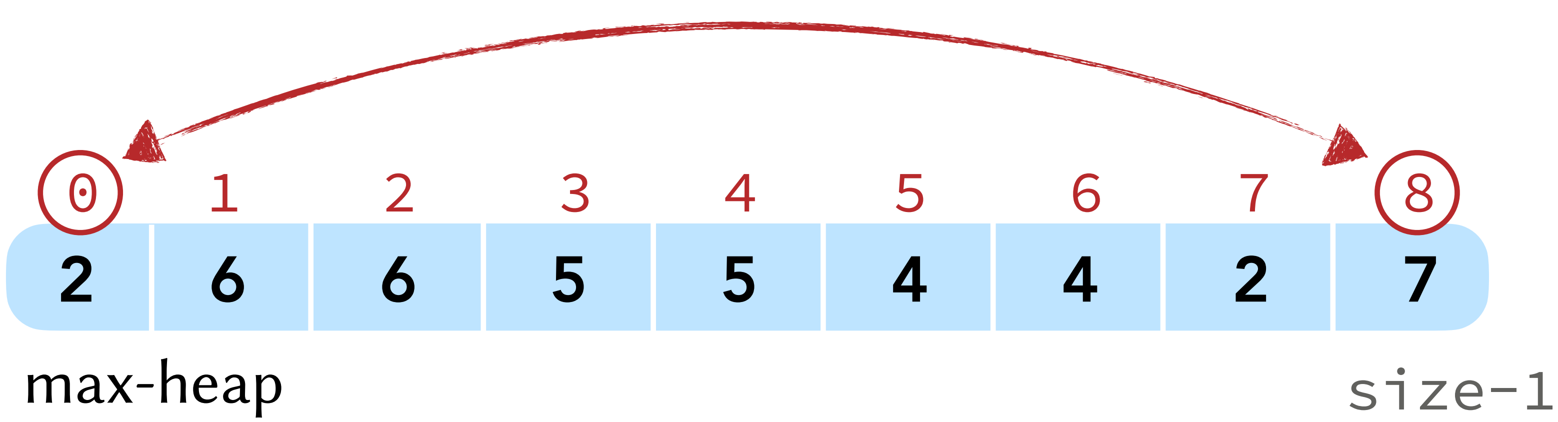

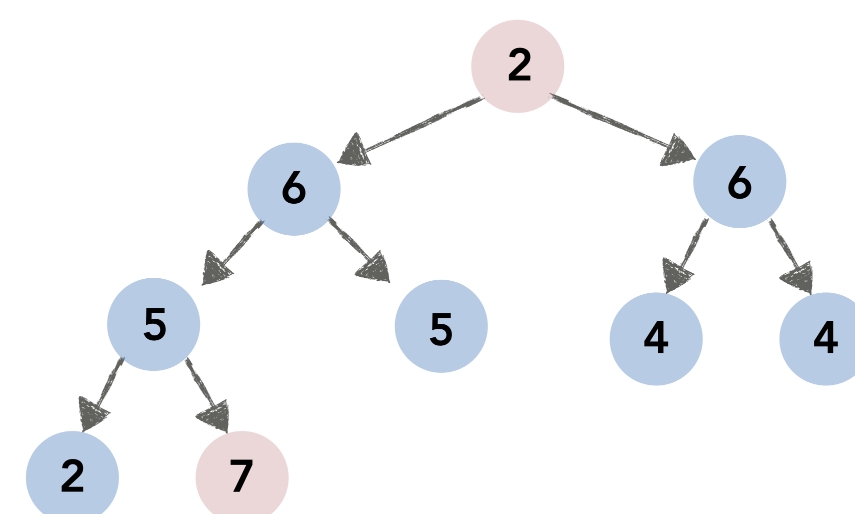

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0	1	2	3	4	5	6	7	8
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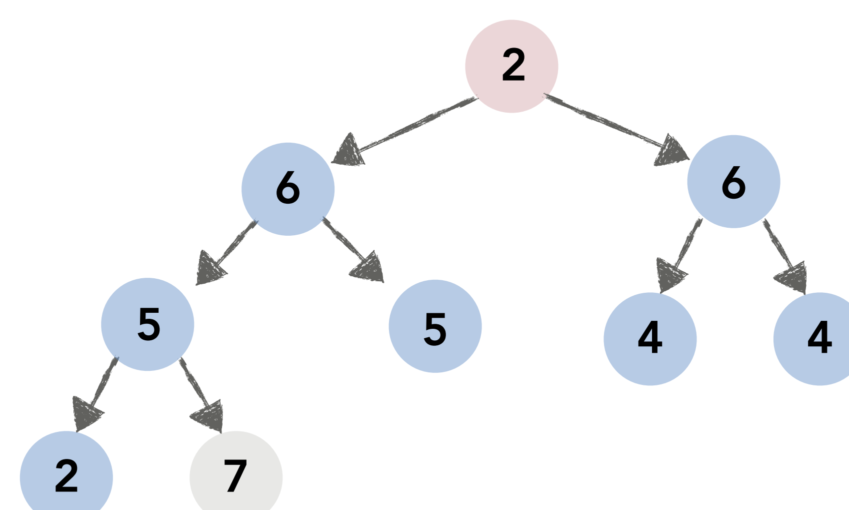
max-heap

size-1



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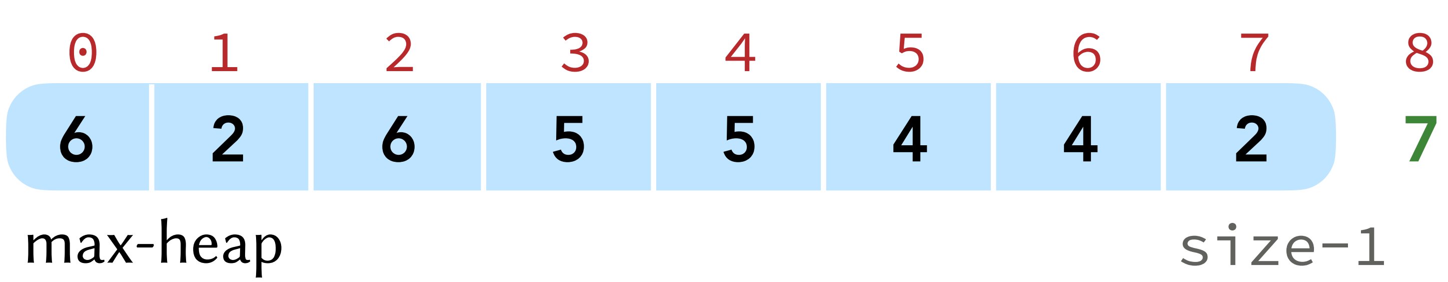

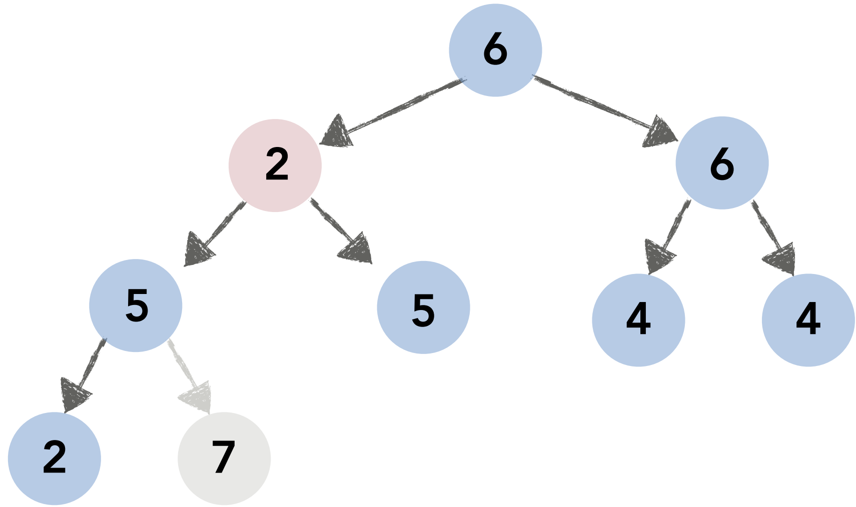

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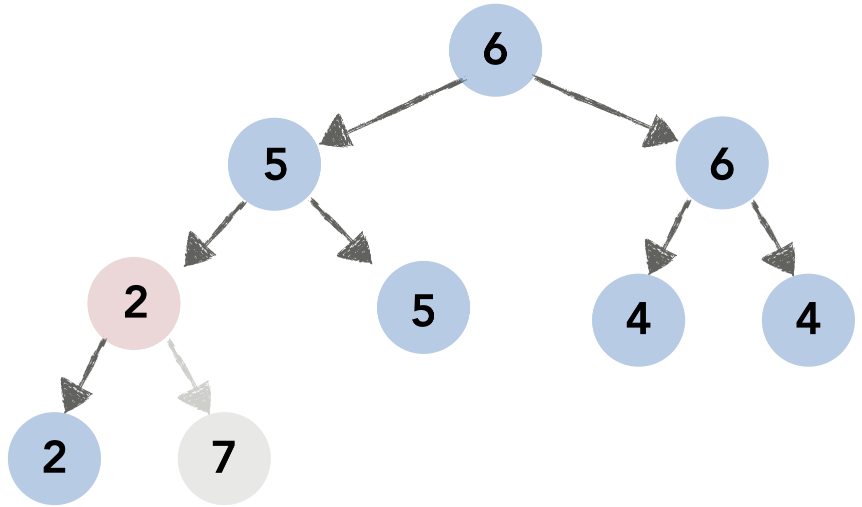
max-heap

size-1



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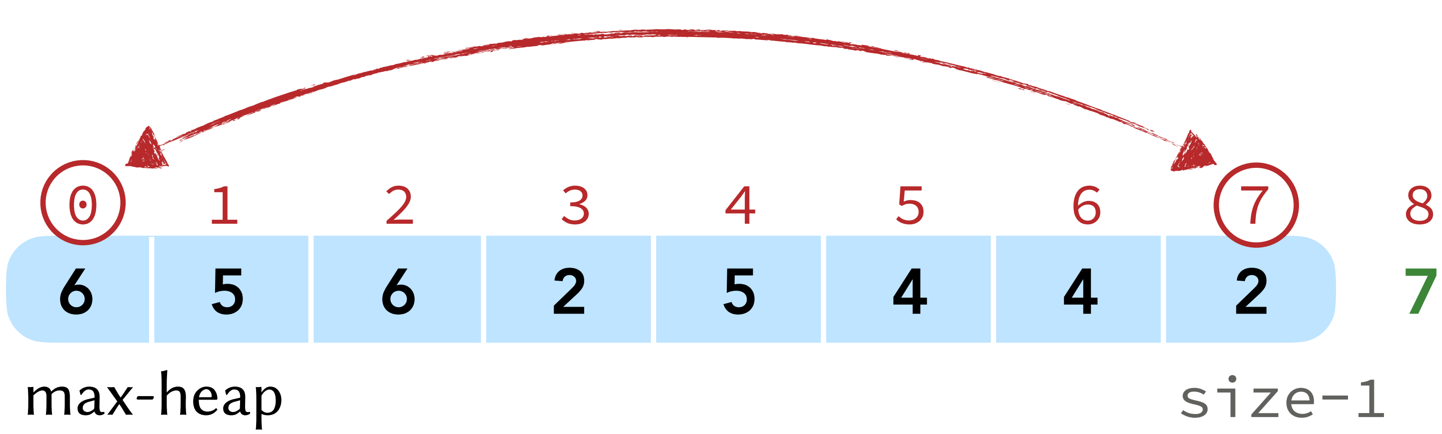

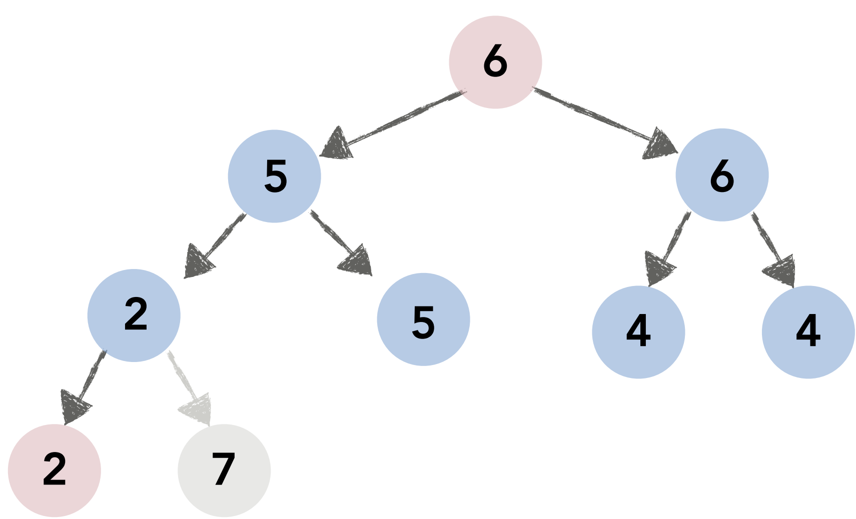

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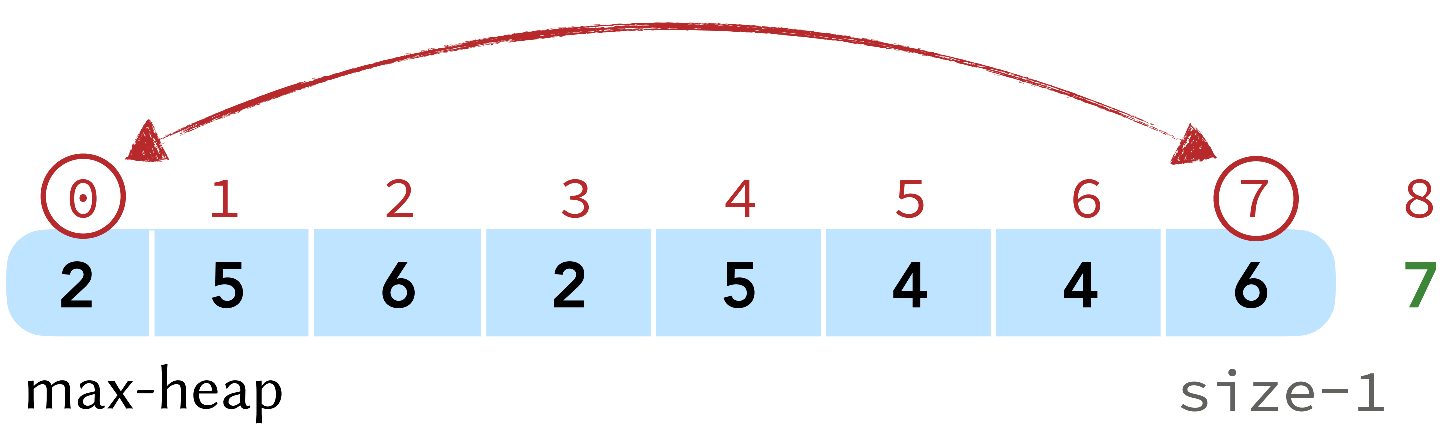

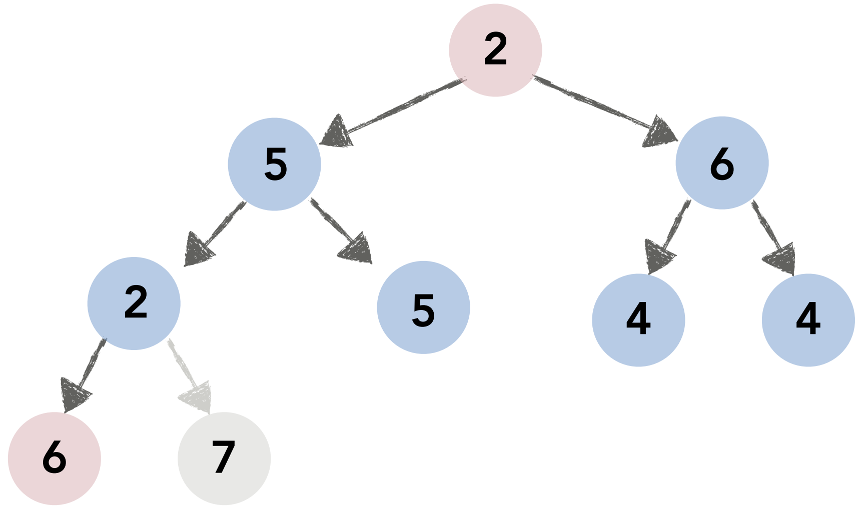

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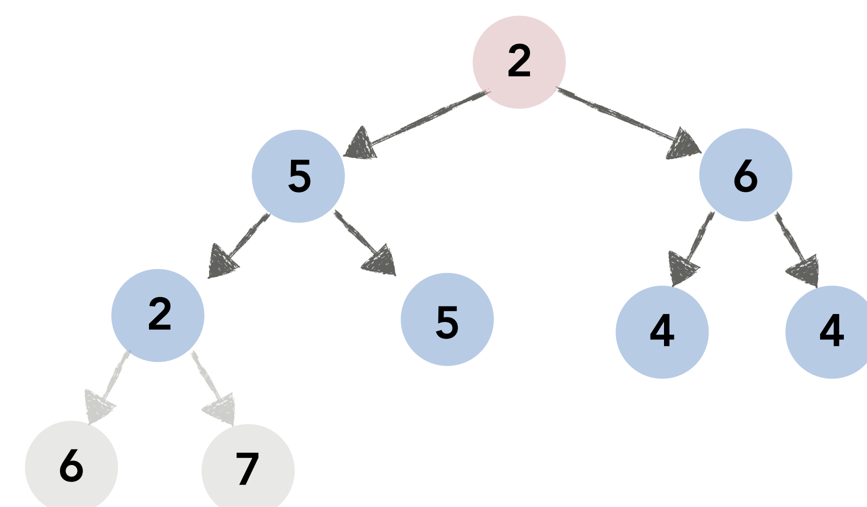
0	1	2	3	4	5	6	7	8
2	5	6	2	5	4	4	6	7

max-heap   size-1



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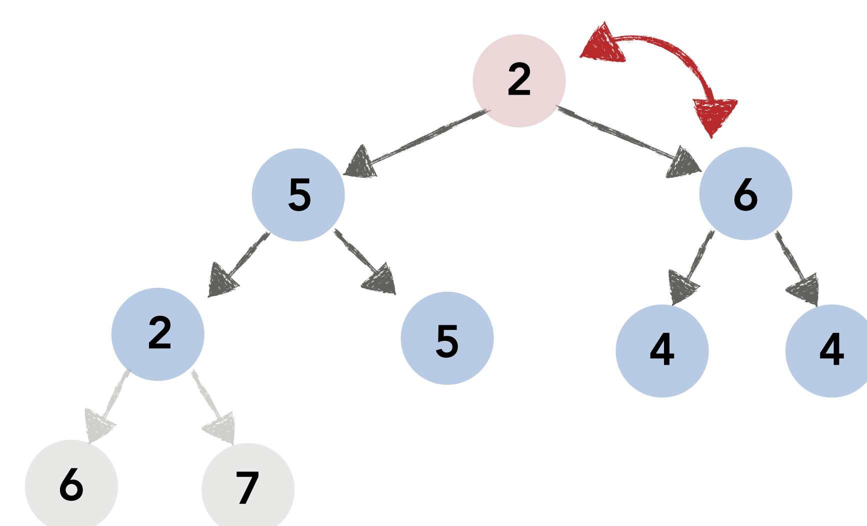

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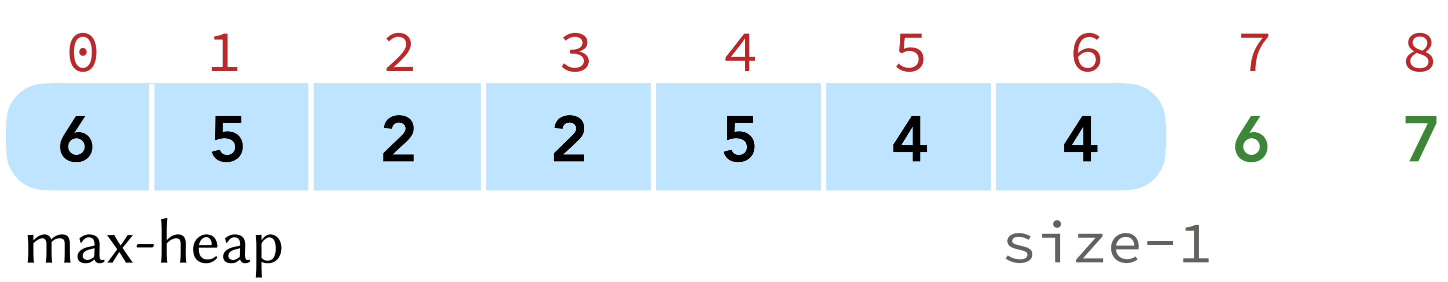

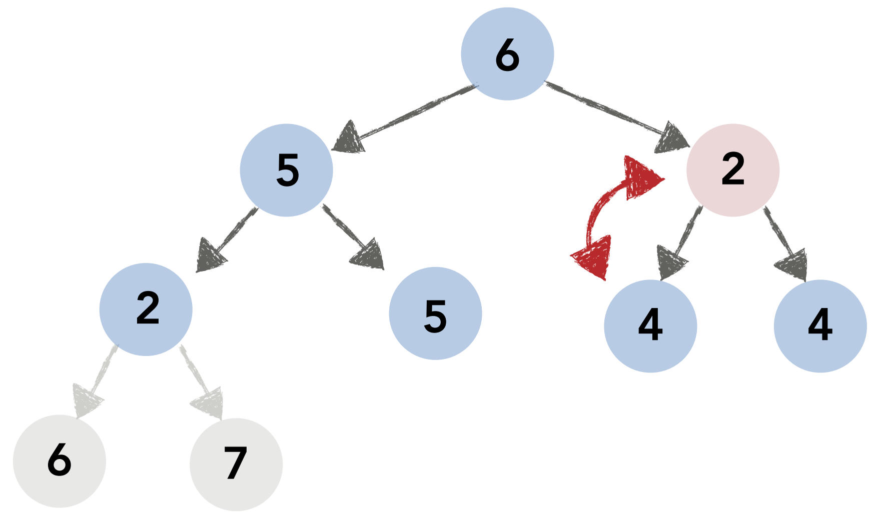

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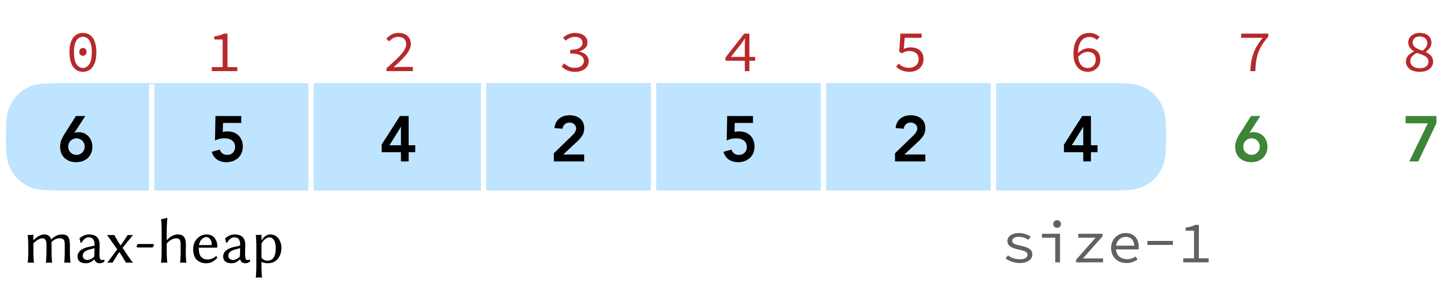

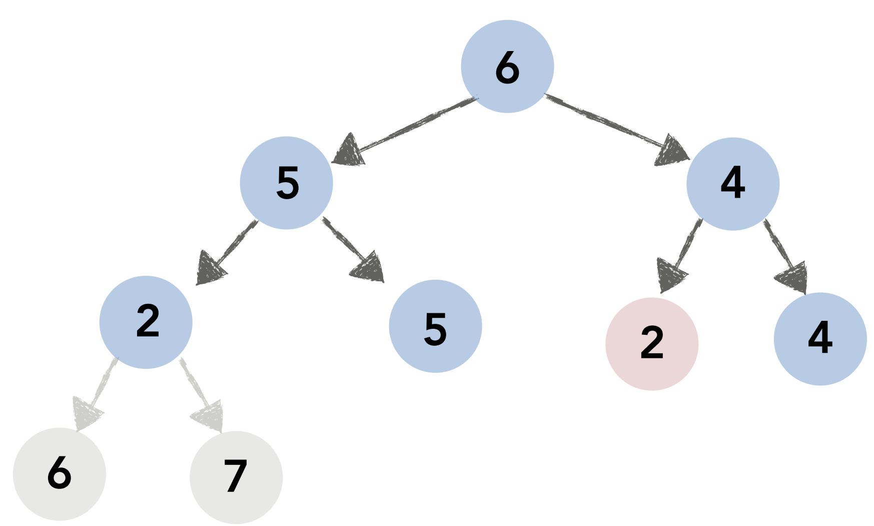

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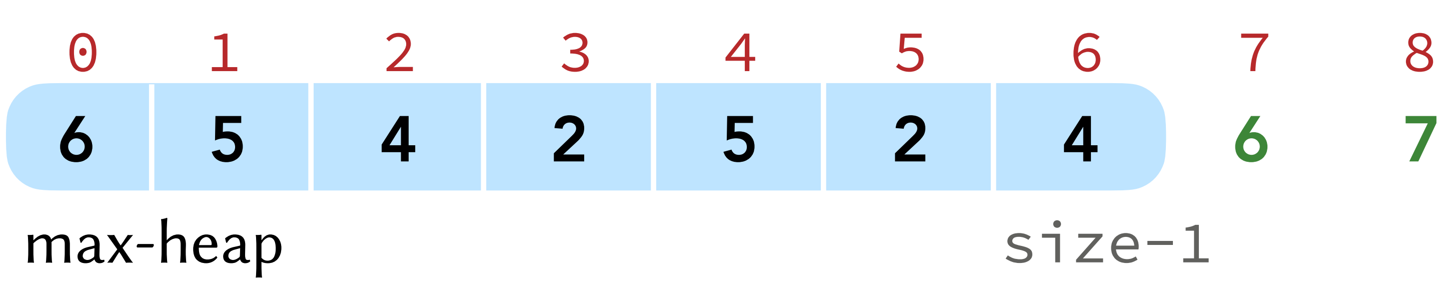
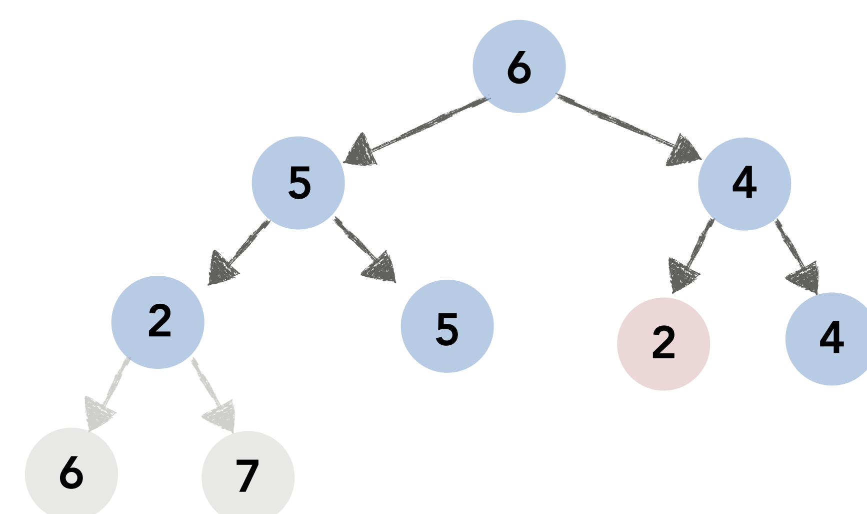
```

1. construct a max-heap in-place (change the array to become a heap)

2. repeatedly place the next maximum in its right position

repeat until all the elements are in their correct positions





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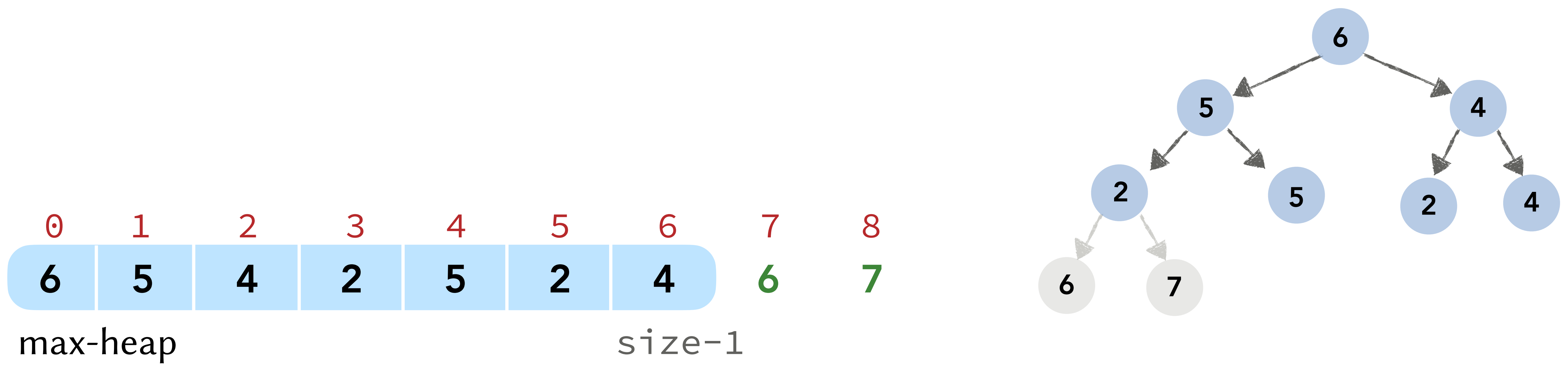

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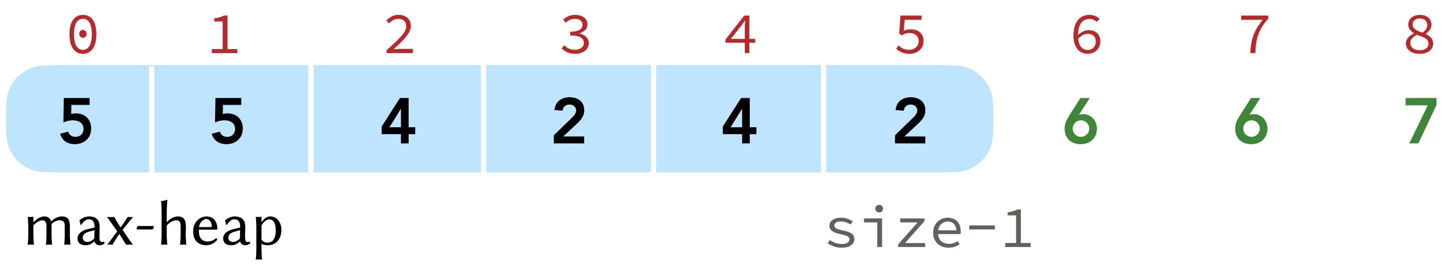

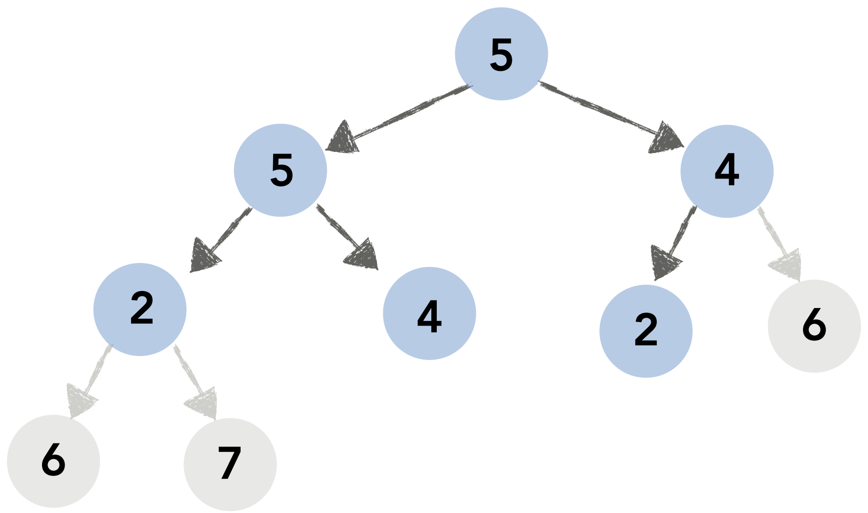

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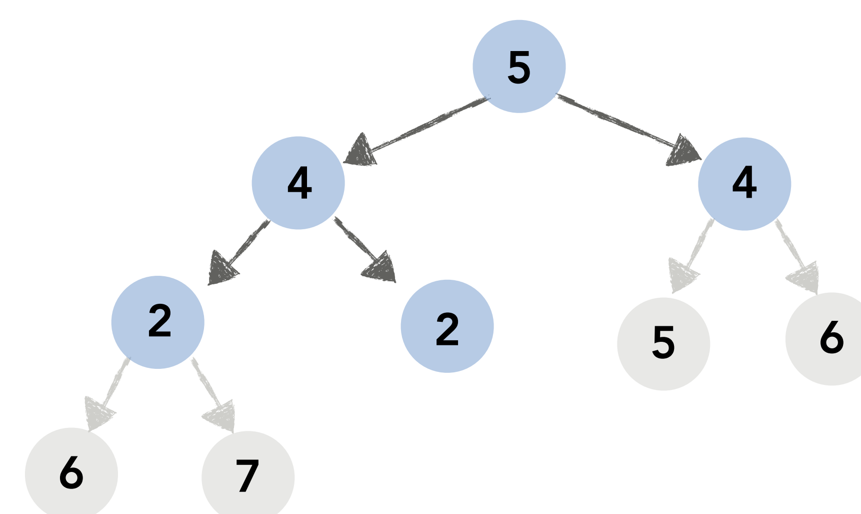
0	1	2	3	4	5	6	7	8
5	4	4	2	2	5	6	6	7

max-heap   size-1



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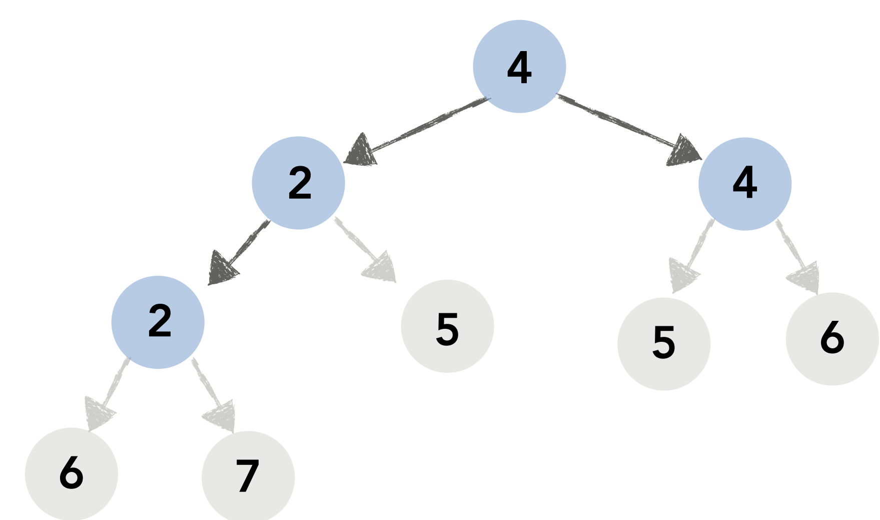
repeat until all the elements are in their correct positions

0	1	2	3	4	5	6	7	8
4	2	4	2	5	5	6	6	7

max-heap   size-1



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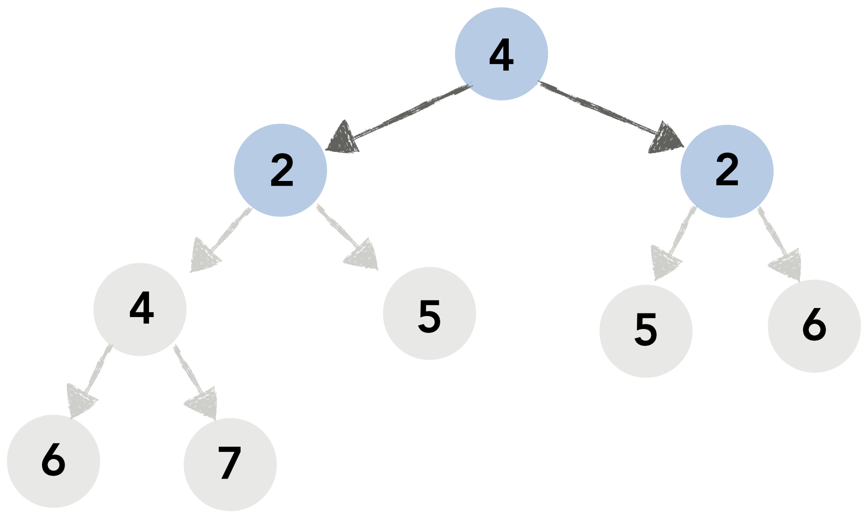
max-heap size-1



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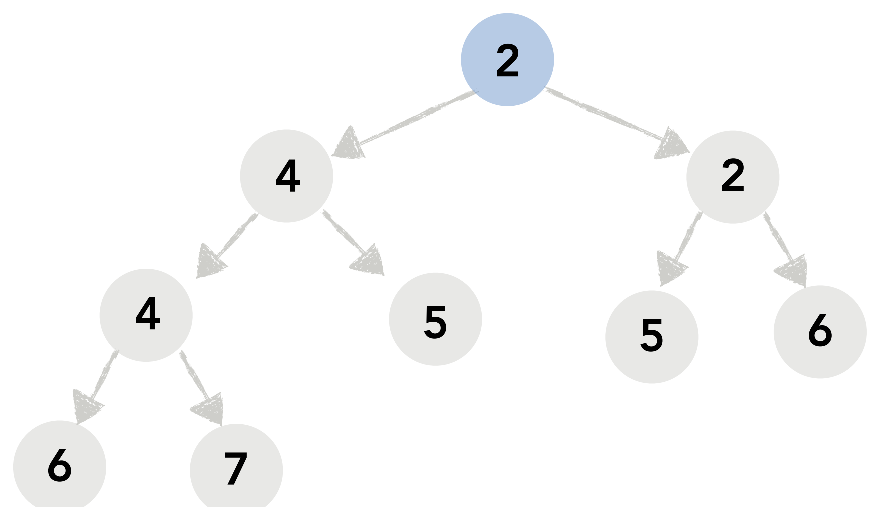
2. repeatedly place the next maximum in its right position

repeat until all the elements are in their correct positions

0	1	2	3	4	5	6	7	8
2	2	4	4	5	5	6	6	7

size-1



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while (size > 1):
    swap(a[0], a[size-1])
    size = size-1
    SINK(a, 0, size)
```

```
CONSTRUCT-HEAP(a, size)

for i = size/2 - 1 → 0:
   SINK(a, i, size)
```

Heapsort: A Better Implementation

```
HEAP-SORT(a[], size)

  CONSTRUCT-HEAP(a, size)

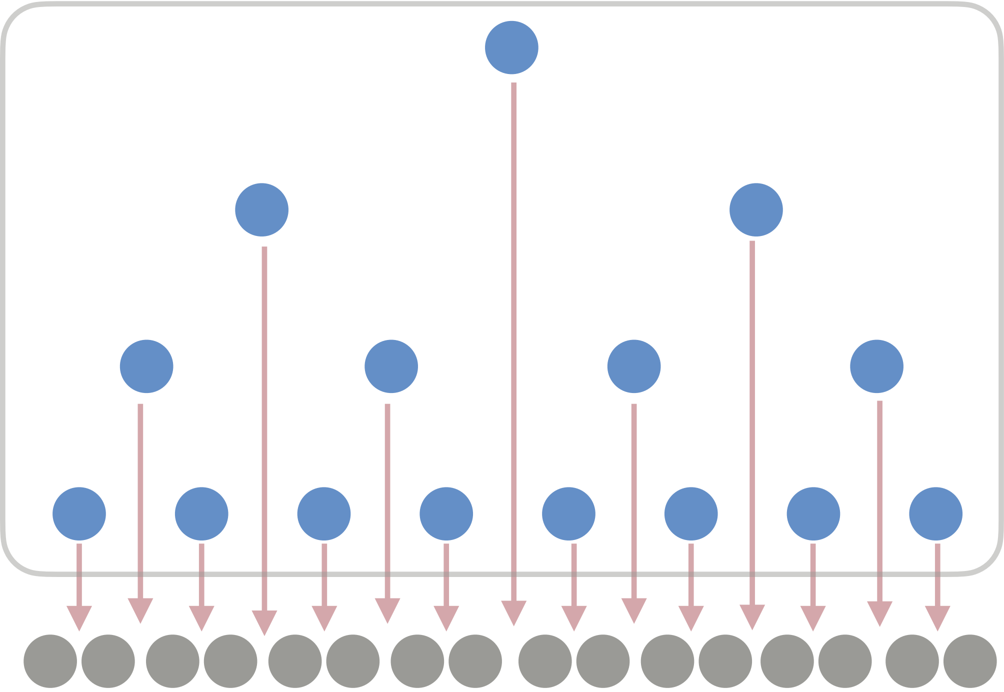
  while (size > 1):
     swap(a[0], a[size-1])
     size = size-1
     SINK(a, 0, size)
```

```
CONSTRUCT-HEAP(a, size)

for i = size/2 - 1 → 0:
   SINK(a, i, size)
```

sink all the non-leaf nodes

at indices 0 to size/2 - 1



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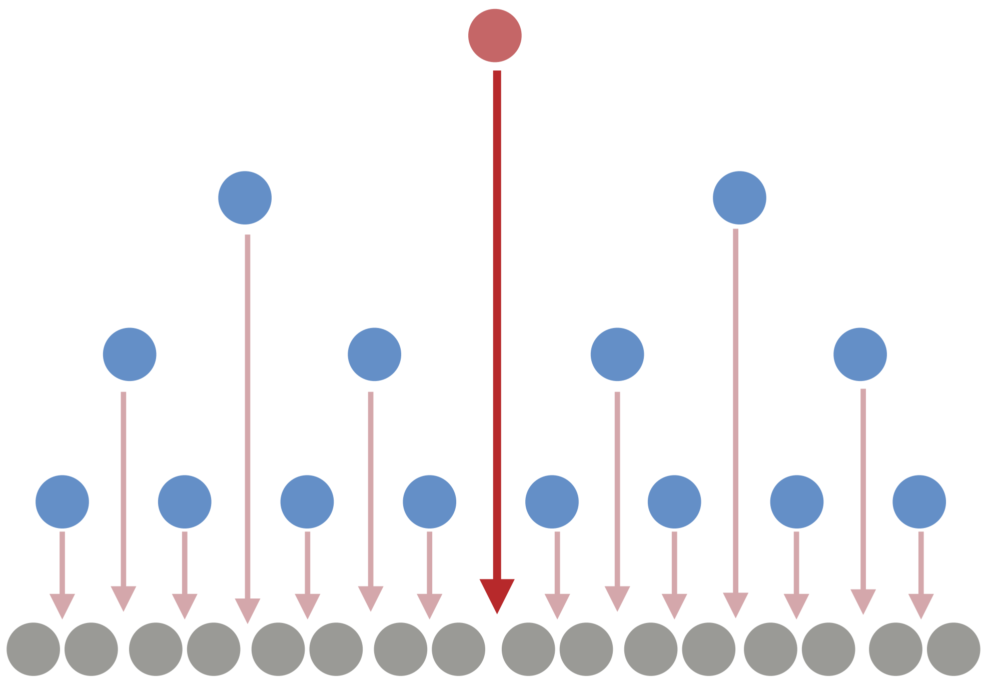
```
CONSTRUCT-HEAP(a, size)

for i = size/2 - 1 → 0:
   SINK(a, i, size)
```



Heap Construction Analysis:

- Maximum number of swaps: $1 \bullet h$ ← number of swaps = tree height

  ↑ number of nodes

Heapsort: A Better Implementation

```
HEAP-SORT(a[], size)

 CONSTRUCT-HEAP(a, size)

 while (size > 1):
     swap(a[0], a[size-1])
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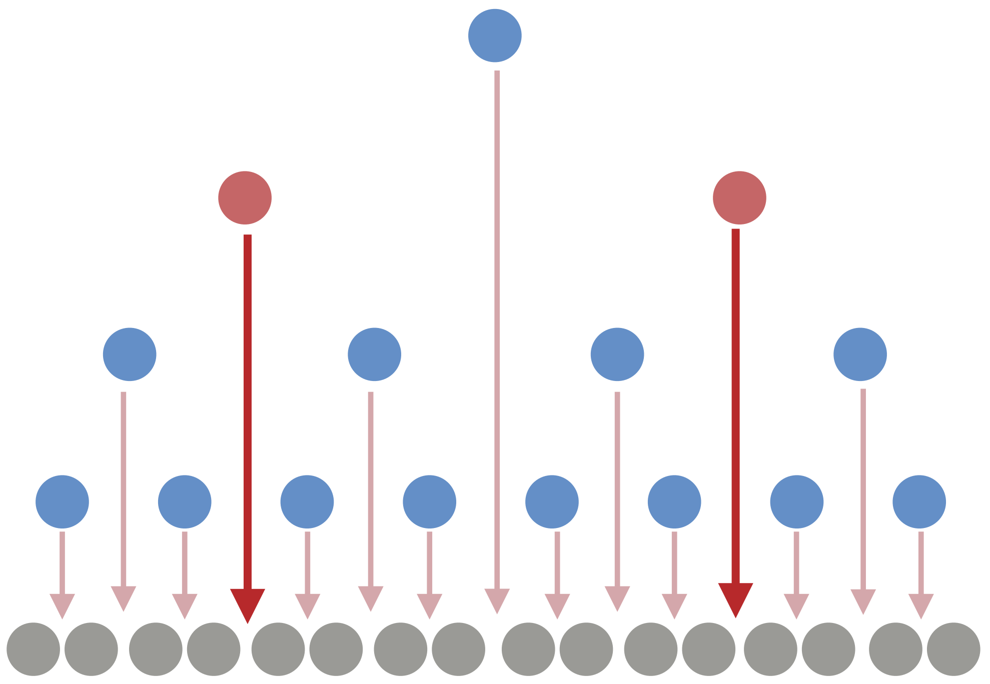

Heap Construction Analysis:

- Maximum number of swaps: $(1 \bullet h) + 2(h - 1)$

Heapsort: A Better Implementation

```
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 CONSTRUCT-HEAP(a, size)

 while (size > 1):
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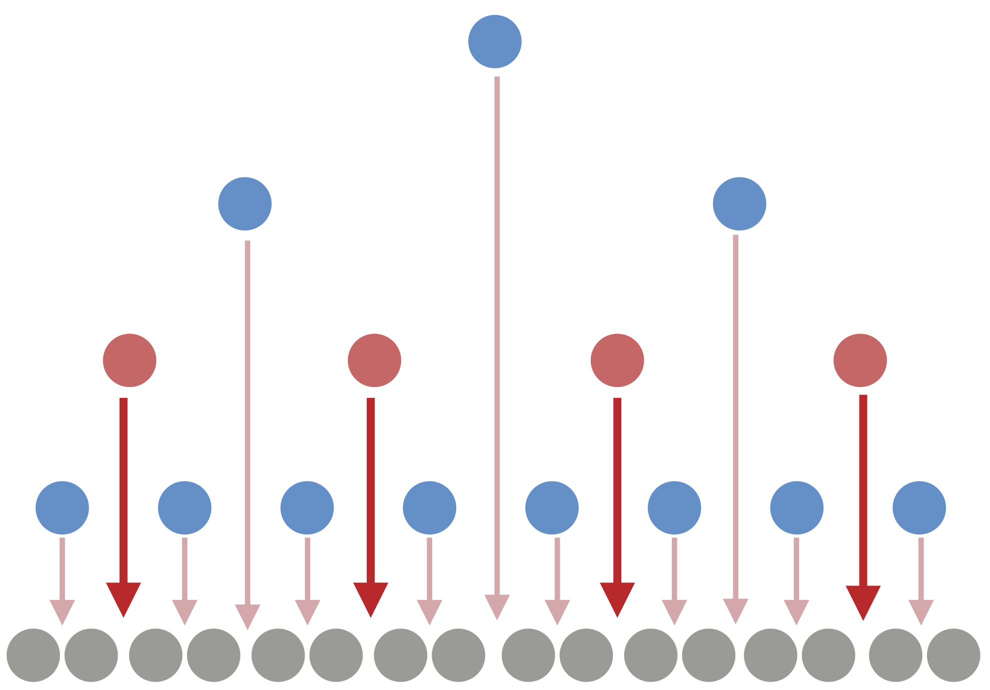

Heap Construction Analysis:

- Maximum number of swaps: $(1 \bullet h) + 2(h-1) + 4(h-2)$

Heapsort: A Better Implementation

```
HEAP-SORT(a[], size)

 CONSTRUCT-HEAP(a, size)

 while (size > 1):
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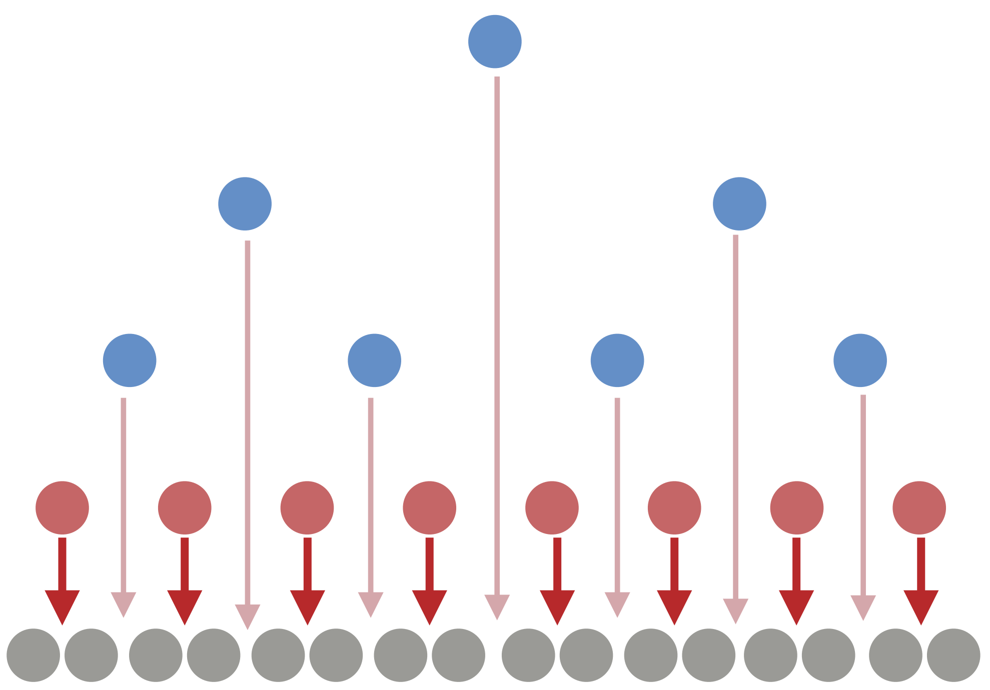

Heap Construction Analysis:

- Maximum number of swaps: $(1 \bullet h) + 2(h-1) + 4(h-2) + \ \ldots \ + (\frac{n}{4} \bullet 1)$

Heapsort: A Better Implementation

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```
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while (size > 1):
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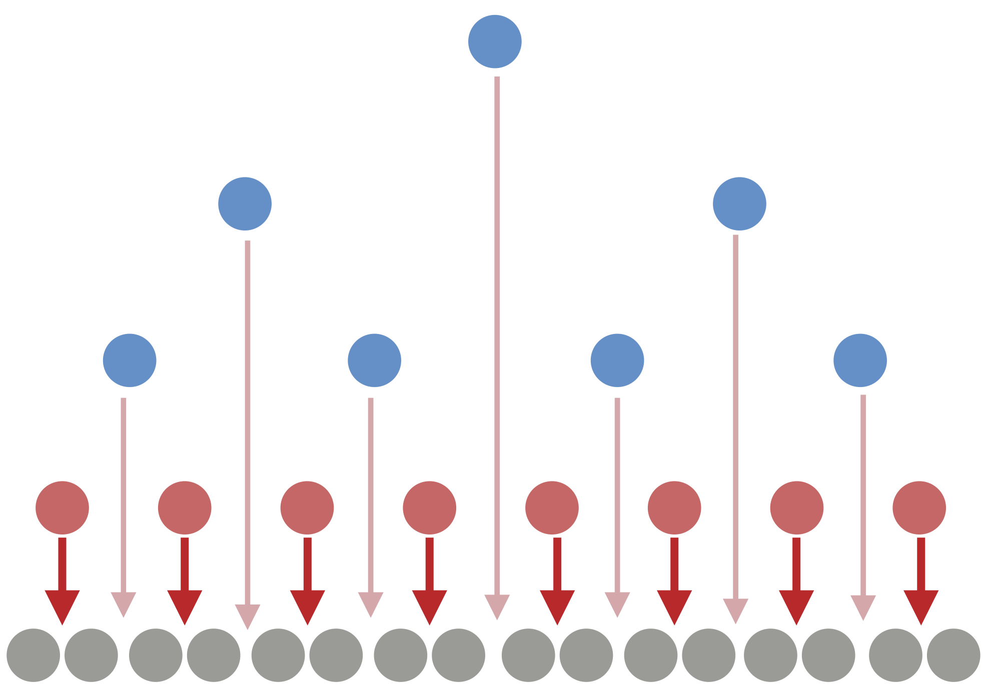

Heap Construction Analysis:

- Maximum number of swaps: $(1 \bullet h) + 2(h-1) + 4(h-2) + \ldots + \frac{n}{4}(1) = O(n)$

tricky sum (math skipped)

Heapsort: A Better Implementation

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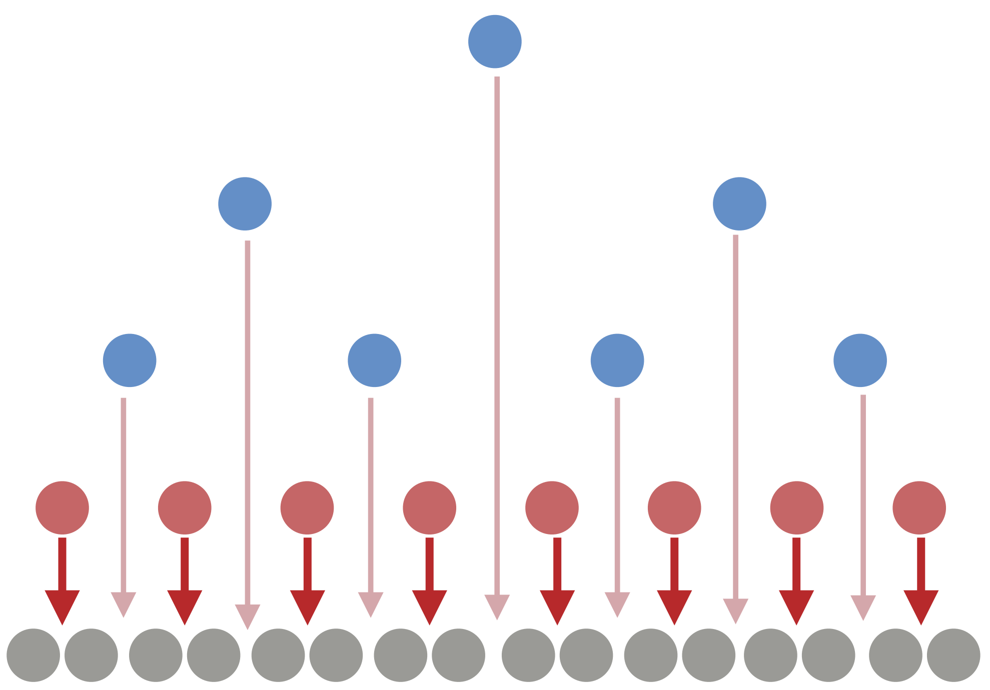

Heap Construction Analysis:

- Maximum number of swaps: $(1 \bullet h) + 2(h-1) + 4(h-2) + \ \dots \ + \frac{n}{4}(1) \quad = O(n)$

number of swaps is linear!

Heapsort: A Better Implementation

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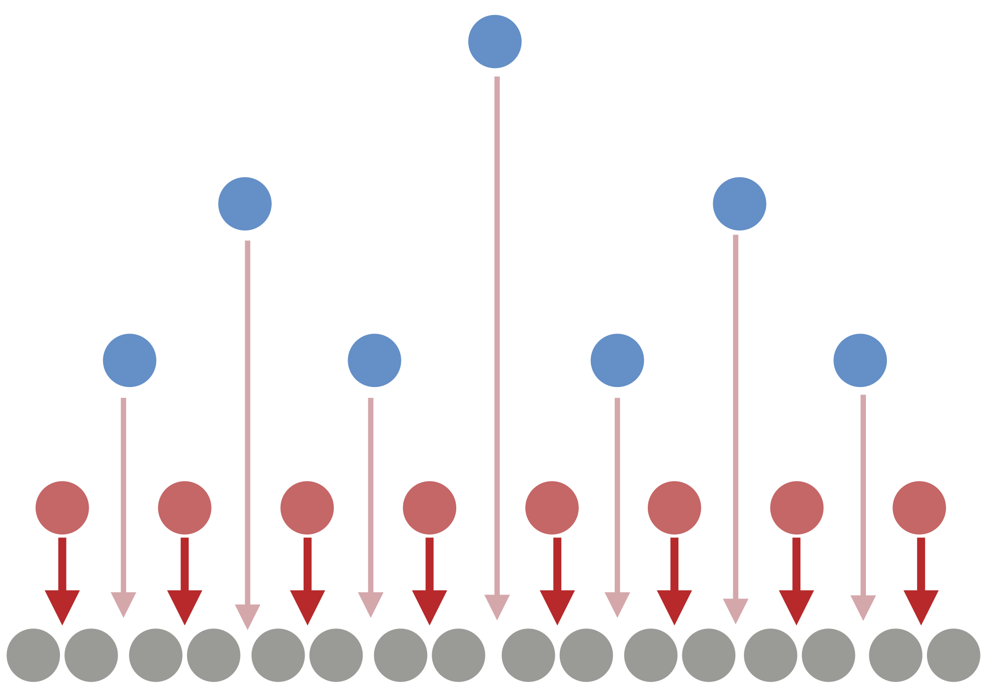

Heap Construction Analysis:

- Maximum number of swaps: $(1 \bullet h) + 2(h-1) + 4(h-2) + \ \ldots \ + \frac{n}{4}(1) \quad = O(n)$
- Maximum number of compares: $2 \times$ number of swaps

check the analysis
of the **SINK** operation!

Heapsort: A Better Implementation

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  CONSTRUCT-HEAP(a, size)

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```

```
CONSTRUCT-HEAP(a, size)

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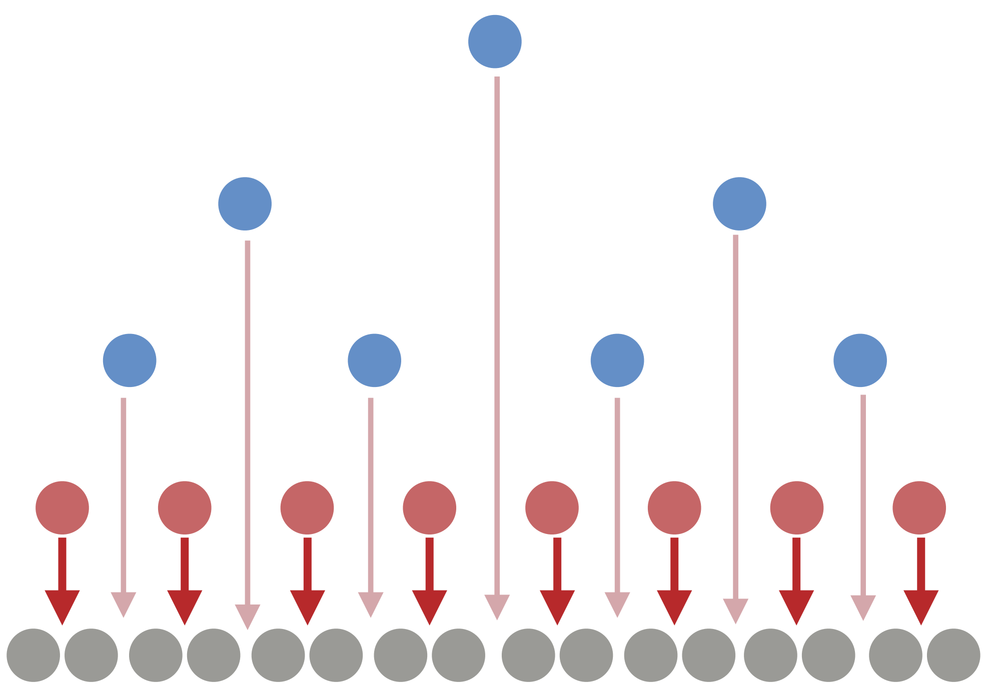
```

Think. Why does this heap construction code run in $O(n)$ while inserting all the elements into a heap takes $O(n \log n)$ time?



Heap Construction Analysis:

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- Maximum number of compares: $2 \times$ number of swaps

check the analysis of the **SINK** operation!

Optional

$$\sum_{i=0}^{n} i \times 2^i = (n-1)2^{n+1} + 2$$

$$(1 \bullet h) + 2(h-1) + 4(h-2) + \ \ldots \ + \ \tfrac{n}{4}(1)$$

$$= 2^0(1 \bullet h) + 2^1(h-1) + 2^2(h-2) + \ \ldots \ + \ 2^{h-1}$$

$$= \sum_{i=0}^{h-1} 2^i(h-i) \ = \ (\sum_{i=0}^{h-1} 2^i h) - (\sum_{i=0}^{h-1} i2^i) \ = \ h(2^h - 1) - (\sum_{i=0}^{h-1} i2^i)$$

$$= h(2^h - 1) - ((h-2)2^h + 2)$$

$$= h(2^h - 1) - (h2^h - 2^{h+1} + 2)$$

$$= h2^h - h - h2^h + 2^{h+1} - 2$$

$$= 2^{h+1} - 2 \quad \longleftarrow \quad h \sim \log_2 n$$

$$= O(n)$$

Heapsort Analysis

Worst Case: $\Theta(n)$ to **construct** the heap and $\Theta(n \log n)$ to **heapsort**.

Average Case: $\Theta(n \log n)$

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Heapsort Analysis

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🤔 **Why?** Trace on a piece of paper to see why!

Heapsort Analysis

Worst Case: $\Theta(n)$ to **construct** the heap and $\Theta(n \log n)$ to **heapsort**.

Average Case: $\Theta(n \log n)$

Best Case: $\Theta(n)$ if all the elements are the same.

Running Time:

- Number of compares: At most $\sim 2n \log_2 n$.

↑

$\sim n \log_2 n$ for merge sort and
$\sim 1.39n \log_2 n$ for quicksort (on random data)

Heapsort Analysis

Worst Case: $\Theta(n)$ to **construct** the heap and $\Theta(n \log n)$ to **heapsort**.

Average Case: $\Theta(n \log n)$

Best Case: $\Theta(n)$ if all the elements are the same.

Running Time:

- Number of compares: At most $\sim 2n \log_2 n$.
- Actual running time: Slower than merge sort and quicksort because of the higher number of comparisons and the the poor use of cache.

optimizations are possible

Heapsort Analysis

Worst Case: $\Theta(n)$ to **construct** the heap and $\Theta(n \log n)$ to **heapsort**.

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Memory. Heapsort is an *in-place* sorting algorithm.

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Memory. Heapsort is an *in-place* sorting algorithm.

Bottom line.

- $\Theta(n \log n)$ in the worst case and also sorts in-place at the same time.
  (Merge Sort is not in-place and Quicksort has a theoretical worst case of $\Theta(n^2)$)
- Practically, not frequently used because it is slower than merge sort and quicksort.

Introsort

From Wikipedia, the free encyclopedia

Introsort or **introspective sort** is a hybrid sorting algorithm that provides both fast average performance and (asymptotically) optimal worst-case performance. It begins with quicksort, it switches to heapsort when the recursion depth exceeds a level based on (the logarithm of) the number of elements being sorted and it switches to insertion sort when the number of elements is below some threshold. This combines the good parts of the three algorithms, with practical performance comparable to quicksort on typical data sets and worst-case $O(n \log n)$ runtime due to the heap sort. Since the three algorithms it uses are comparison sorts, it is also a comparison sort.

Introsort

Class	Sorting algorithm
Data structure	Array
Worst-case performance	$O(n \log n)$
Average performance	$O(n \log n)$

Used for the C++ STL `sort()` function

Sorting algorithms: summary

	inplace?	stable?	best	average	worst	remarks
selection	✔		$\frac{1}{2} n^2$	$\frac{1}{2} n^2$	$\frac{1}{2} n^2$	n exchanges
insertion	✔	✔	n	$\frac{1}{4} n^2$	$\frac{1}{2} n^2$	use for small n or partially ordered
merge		✔	$\frac{1}{2} n \log_2 n$	$n \log_2 n$	$n \log_2 n$	$\Theta(n \log n)$ guarantee; stable
timsort		✔	n	$n \log_2 n$	$n \log_2 n$	improves mergesort when pre-existing order
quick	✔		$n \log_2 n$	$2 n \ln n$	$\frac{1}{2} n^2$	$\Theta(n \log n)$ probabilistic guarantee; fastest in practice
3-way quick	✔		n	$2 n \ln n$	$\frac{1}{2} n^2$	improves quicksort when duplicate keys
heap	✔		$3 n$	$2 n \log_2 n$	$2 n \log_2 n$	$\Theta(n \log n)$ guarantee; in-place
?	✔	✔	n	$n \log_2 n$	$n \log_2 n$	holy sorting grail

number of compares to sort an array of n elements

By Kevin Wayne